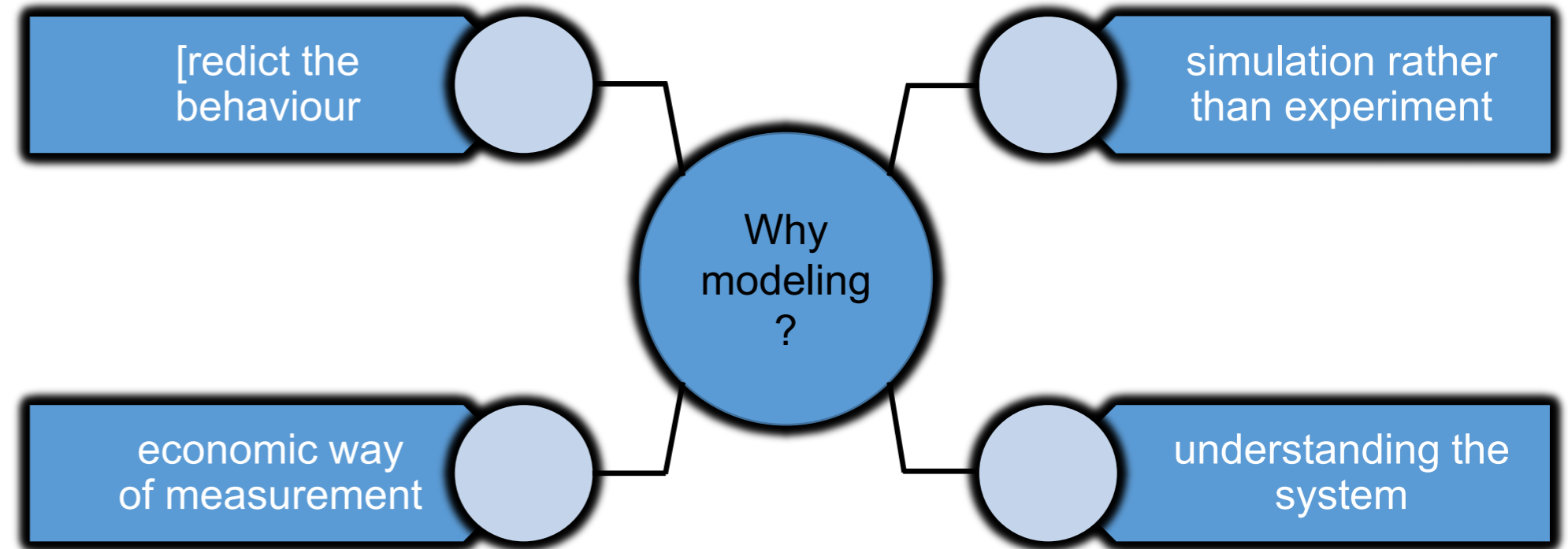
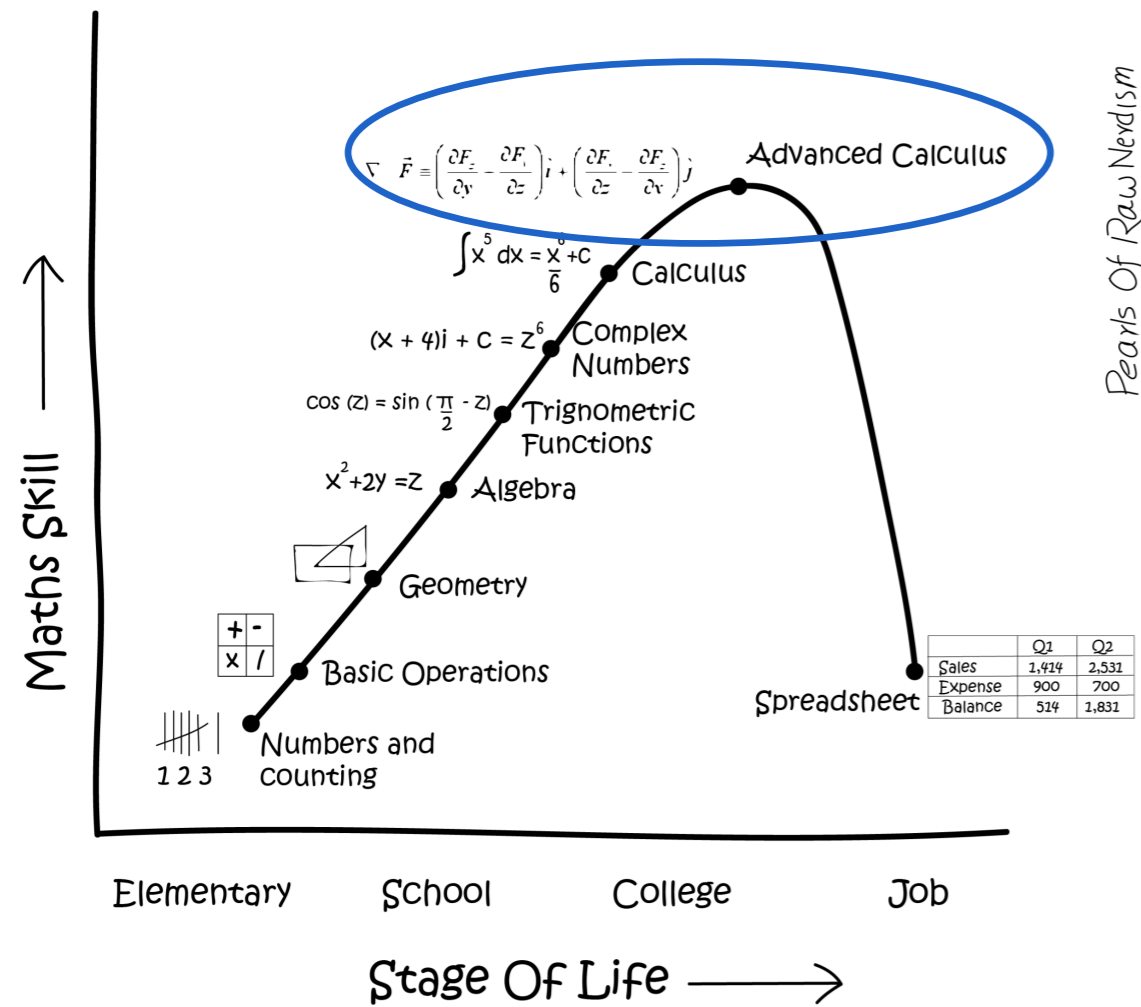


Parameter Estimation Using Python

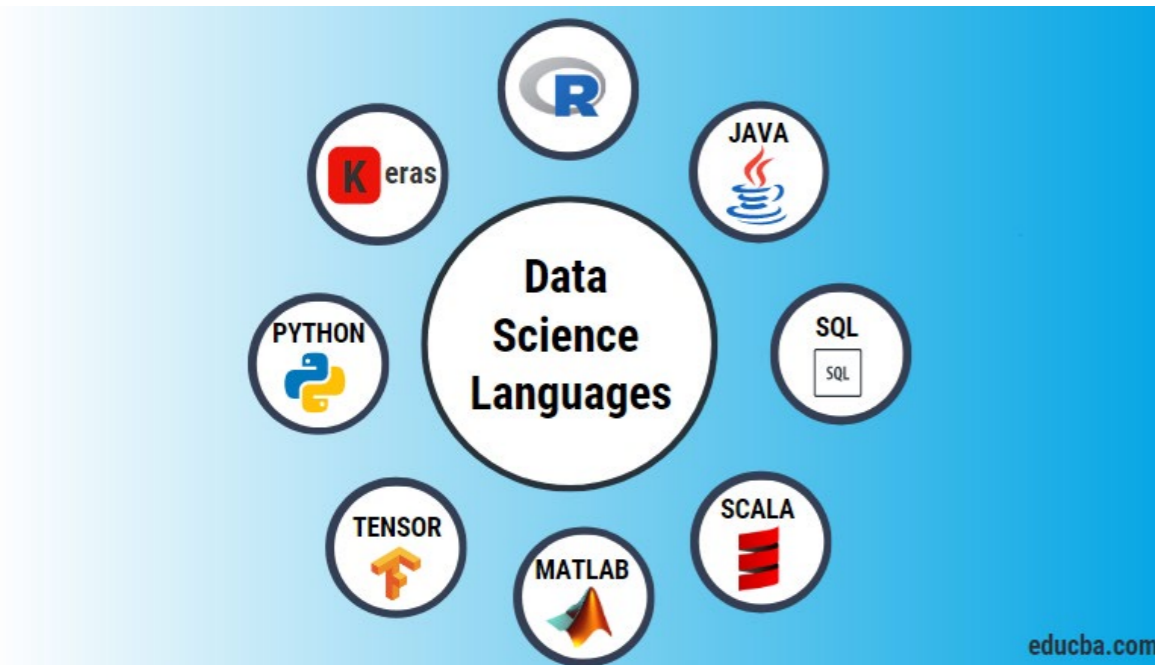
Reza Monjezi¹, Javier Ibanez Abad¹, Ana Bjelic¹, Joris W. Thybaut¹

¹Laboratory for Chemical Technology

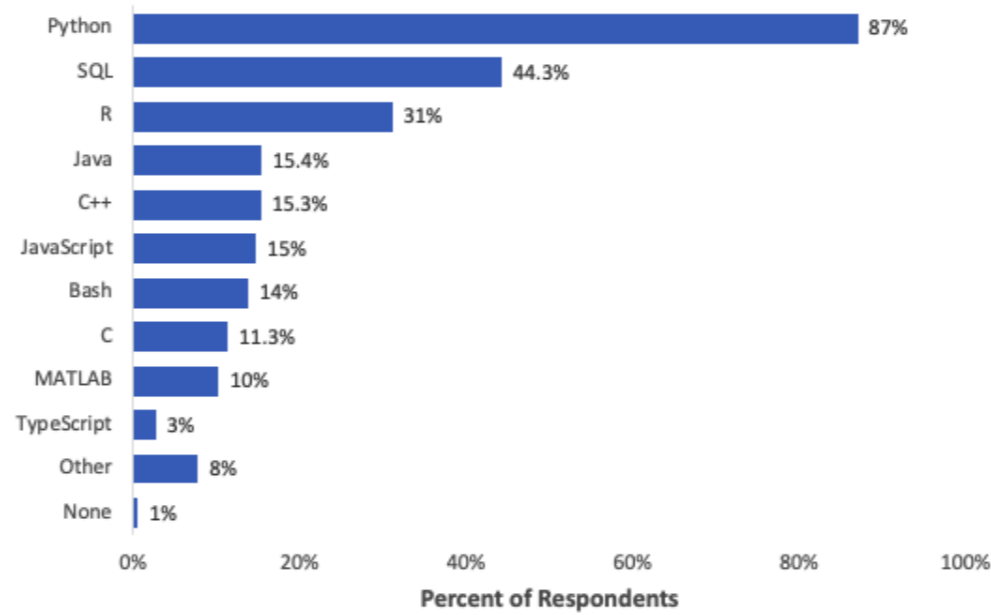
importance of mathematical modelling



programming language



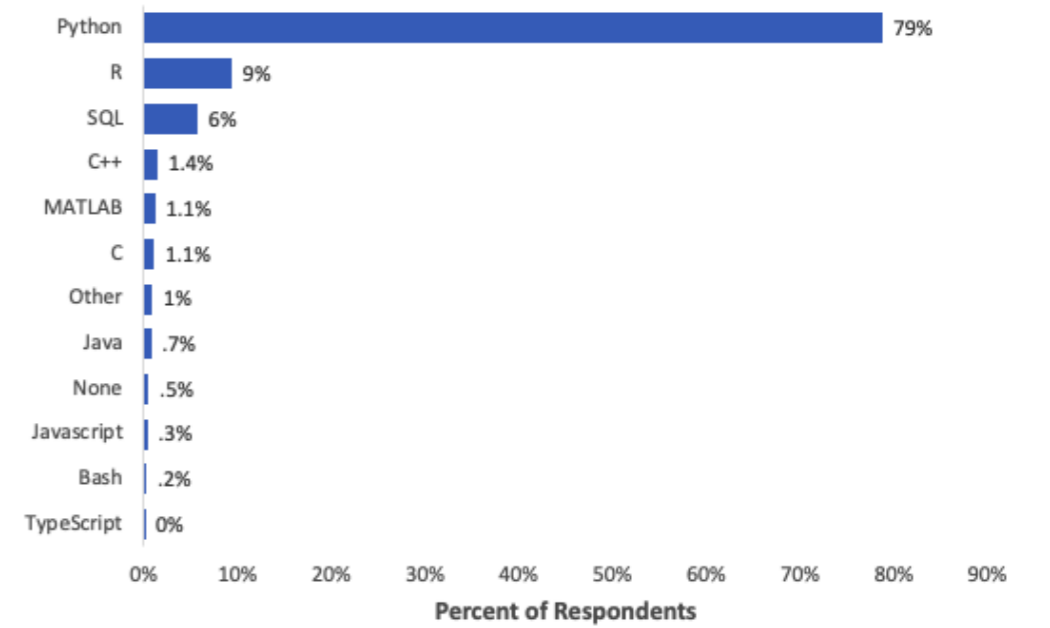
What programming languages do you use on a regular basis?



Note: Data are from the 2019 Kaggle ML and Data Science Survey. You can learn more about the study here: <https://www.kaggle.com/c/kaggle-survey-2019>. A total of 19717 respondents completed the survey; the percentages in the graph are based on a total of 14762 respondents who provided an answer to this question.



What programming language would you recommend an aspiring data scientist to learn first?



Note: Data are from the 2018 Kaggle ML and Data Science Survey. You can learn more about the study here: <https://www.kaggle.com/c/kaggle-survey-2019>. A total of 19717 respondents completed the survey; the percentages in the graph are based on a total of 14377 respondents who provided an answer to this question.

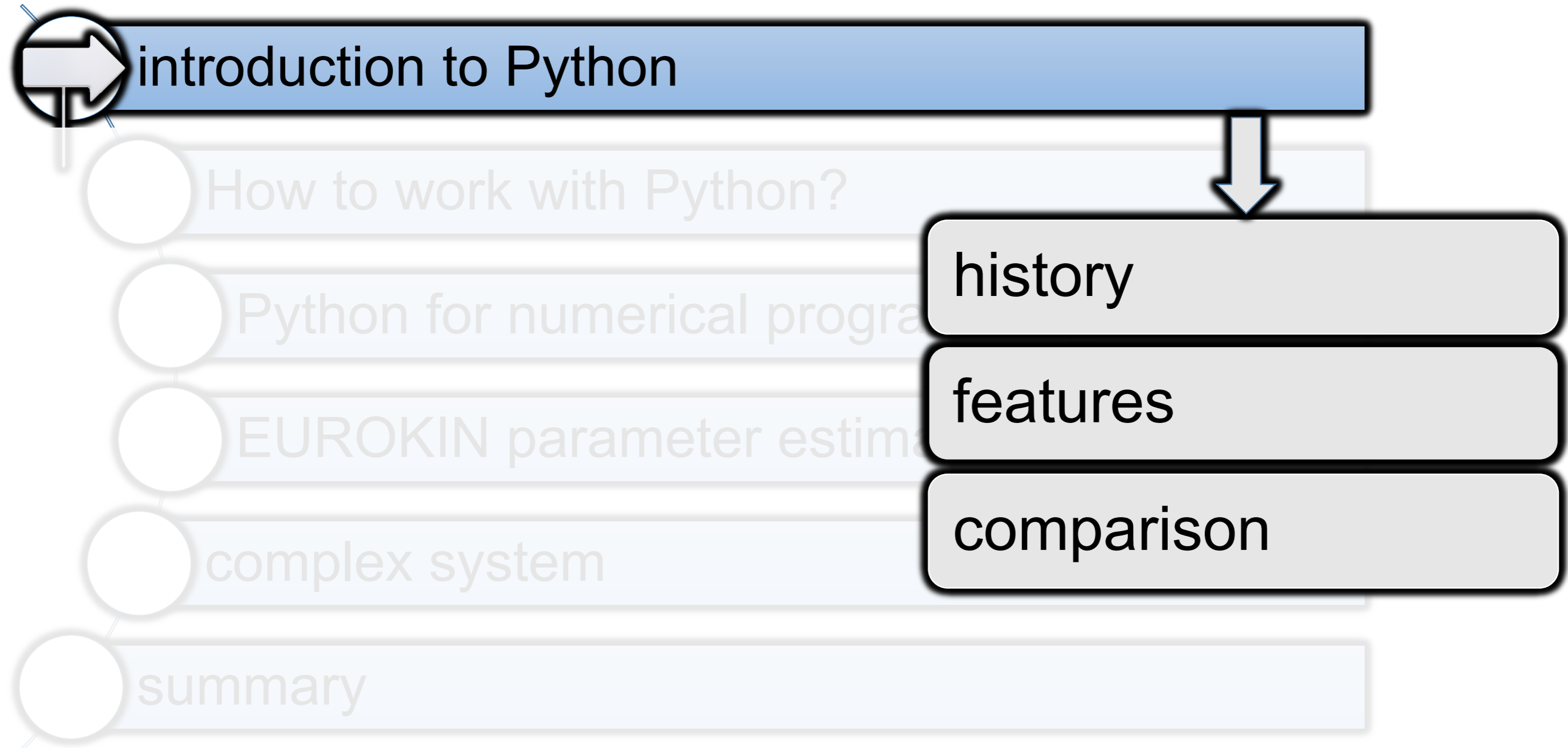


programming language



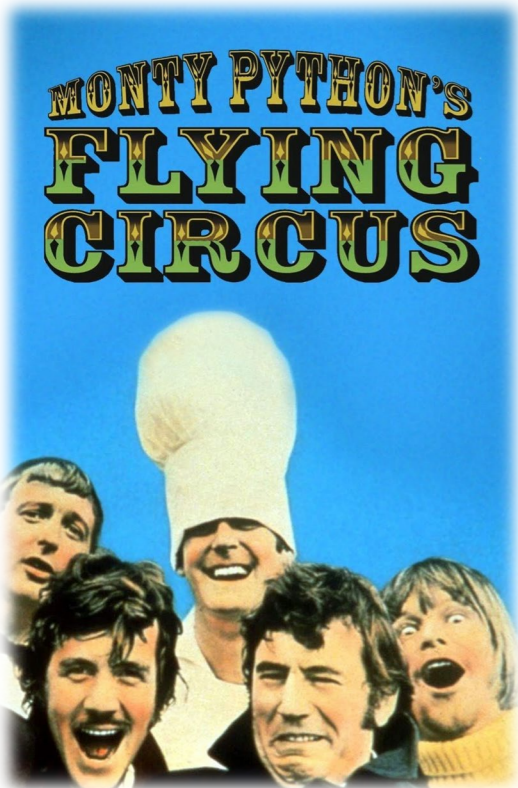
python

overview



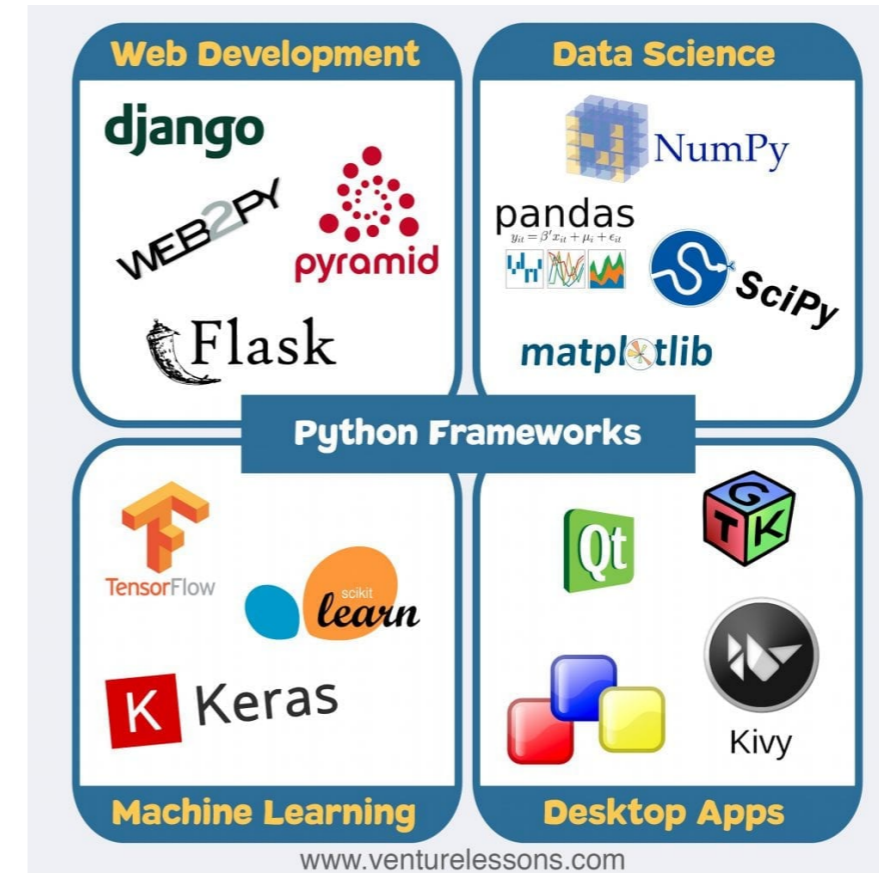
introduction to Python

Guido van Rossum

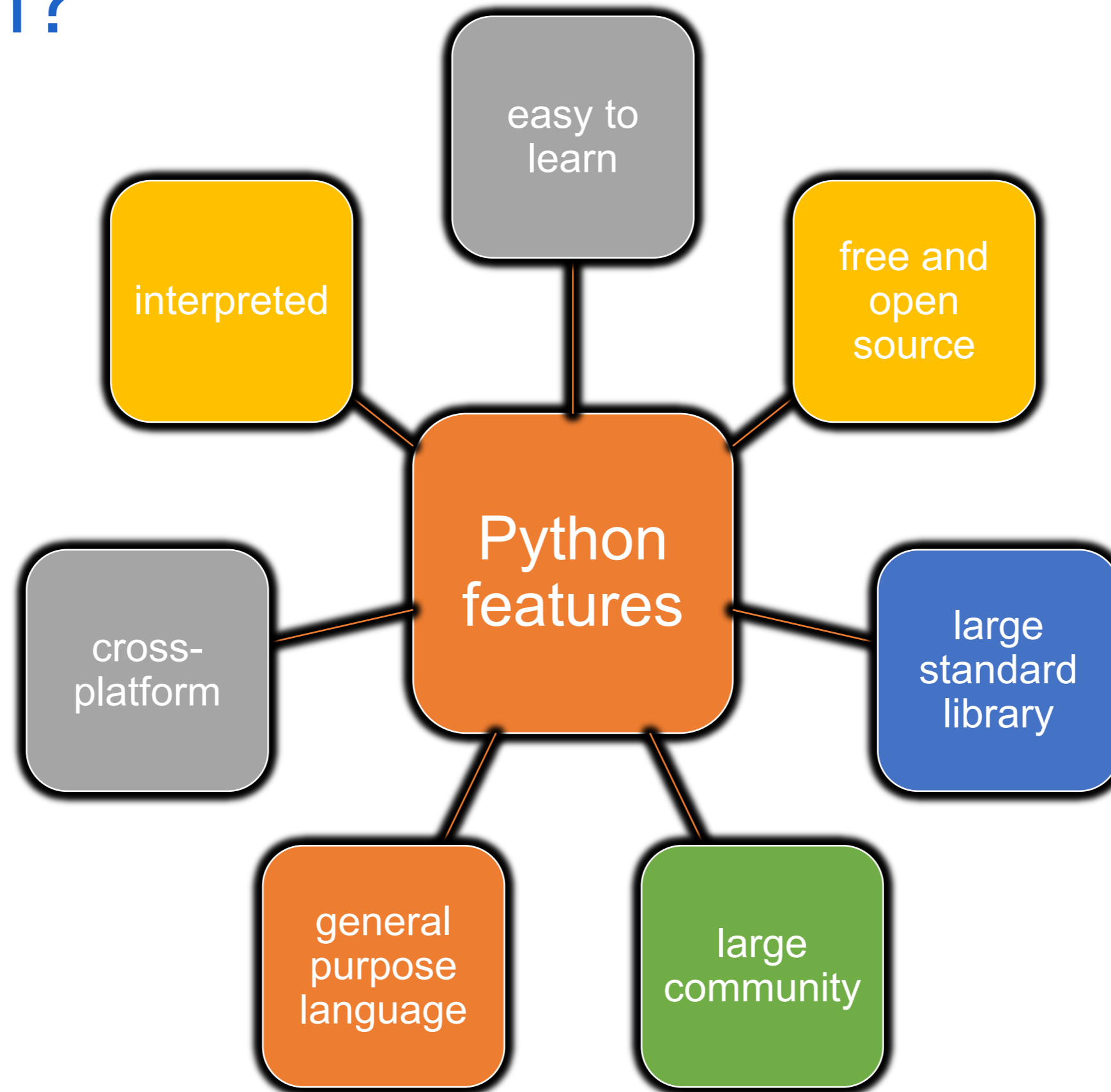


one of the fastest growing programming languages in terms of

- No. of developers
- No. of libraries
- No. of areas
- No. of companies



Why Python?

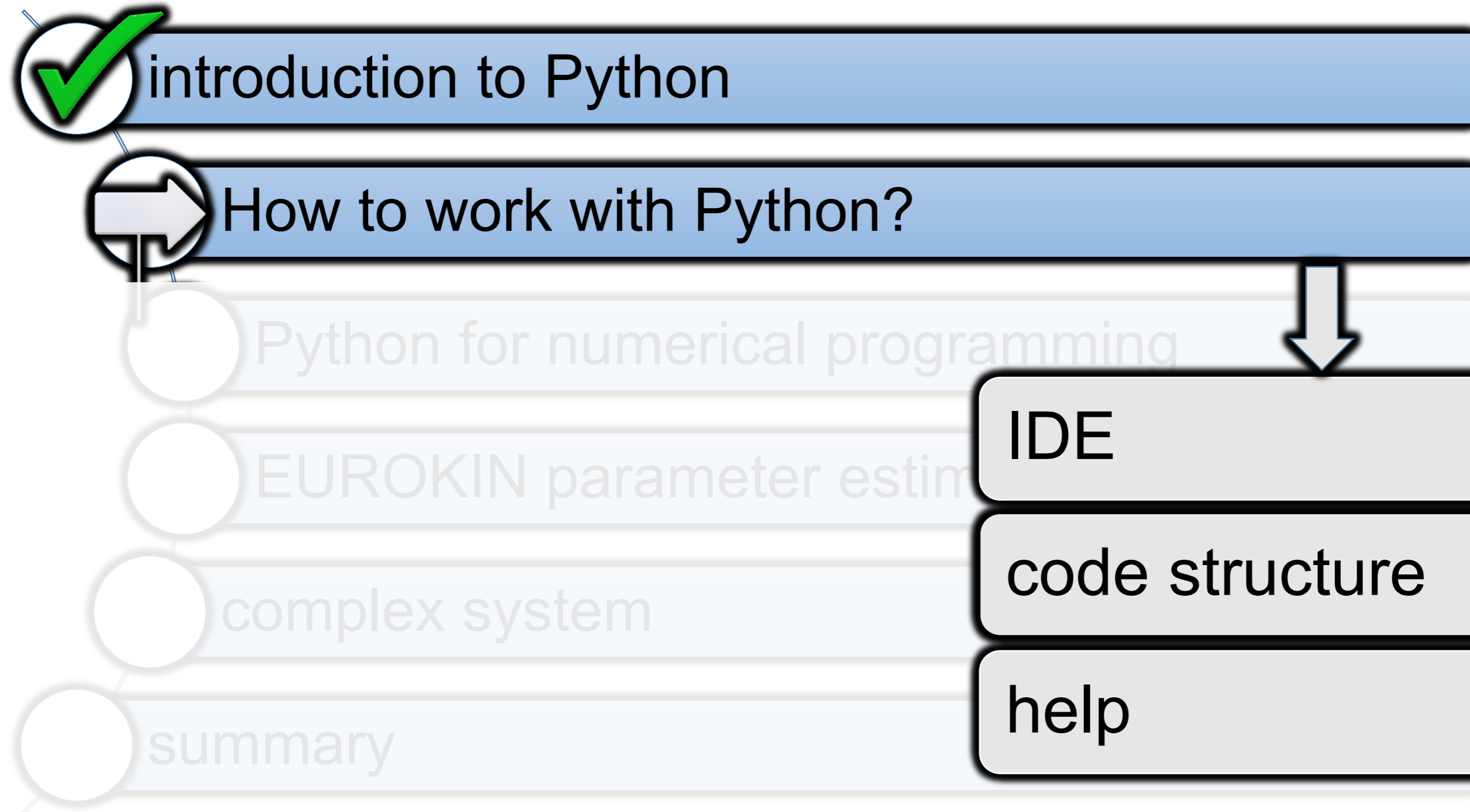


Python vs. Fortran and Matlab

	Python	Fortran	Matlab	
free?	✓	✓	✓	✗
open source?	✓	✗	✗	✗
interpreted?	✓	✓	✓	✓
all in one?	✓	✓	✓	✓
support from company?	✓	✓	✓	✓
easy import/export data?	✓	✓	✓	✓
easy data visualization?	✓	✓	✓	✓



overview



IDEs for Python



IDEs for Python

The screenshot displays the Spyder Python IDE interface. The main window is titled "Spyder (Python 3.7)" and contains a menu bar (File, Edit, Search, Source, Run, Debug, Consoles, Projects, Tools, View, Help) and a toolbar. The file explorer shows the current project path: "...\\WORK\\PhD Project\\Modeling\\EUROKIN\\Parameter estimation-Python\\Problem2_EUROKIN\\Par_est_problem_2.py". The code editor contains the following Python code:

```
1 #Import all the packages
2 import numpy as np
3 from scipy.integrate import odeint
4 from scipy.integrate import solve_ivp
5 import matplotlib.pyplot as plt
6 import pandas as pd
7 from scipy.optimize import curve_fit
8 from scipy.stats import probplot
9 from scipy.stats import f
10 from scipy.stats import t as t_test
11 from uncertainties import ufloat
12 from scipy.optimize import fsolve
13 import sys
14 from scipy.optimize import root
15
16
17
18 #Import residence time. res_t_All means all the residence times for isothermal
19 # and nonisothermal cases. but only 8 of them are at T=330K
20 data_file = pd.read_excel('Data_2.xlsx')
21 rl=data_file.loc[0:12,'Reactor length']
22 rl=pd.DataFrame(rl).to_numpy()
23
24
25 C0=data_file.loc[0:12,'Cb0':'Cd0']
26 C0=pd.DataFrame(C0).to_numpy()
27
28
29 C=data_file.loc[0:12,'Ca':'Cd']
30 C=pd.DataFrame(C).to_numpy()
31 C0=np.c_[C[:,0],C0]
32
33 # =====
34
35 #Solving non linear systems of equations
36 def rxn_Cs(Cs_sol,k1,k2,k1m,k2m,k3m,Ca,Cb,Cc):
37
38     Cas,Cbs=Cs_sol
39
40     k1=np.exp(k1)
41     k2=np.exp(k2)
42     k1m=np.exp(k1m)
43     k2m=np.exp(k2m)
```

The right sidebar contains a "Usage" help window with the following text:

Here you can get help of any object by pressing **Ctrl+I** in front of it, either on the Editor or the Console.

Help can also be shown automatically after writing a left parenthesis next to an object. You can activate this behavior in *Preferences > Help*.

New to Spyder? Read our [tutorial](#)

Below the help window is the IPython console, which shows the following output:

```
Python 3.7.9 (default, Aug 31 2020, 17:10:11) [MSC v.1916 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 7.19.0 -- An enhanced Interactive Python.

In [1]:
```

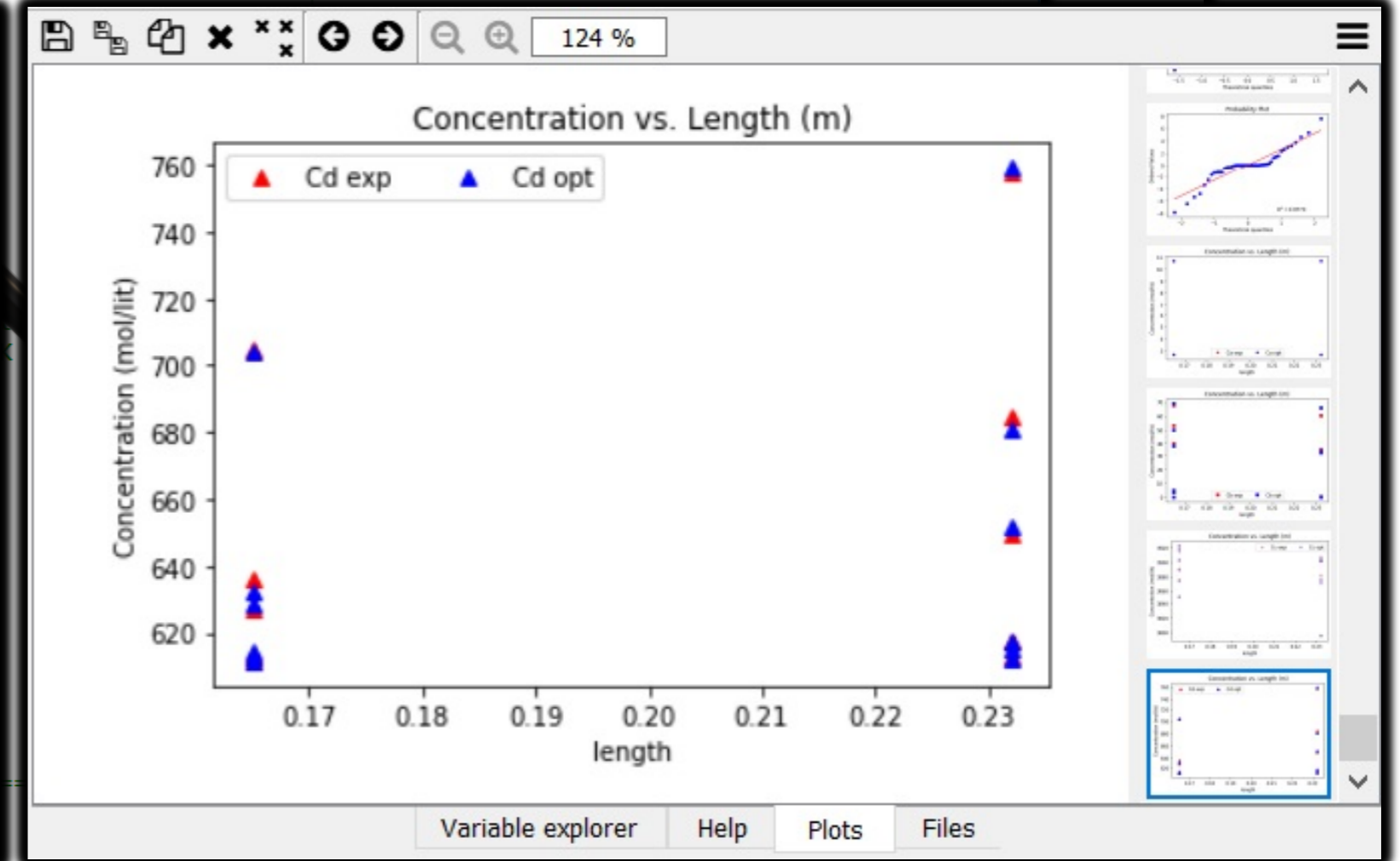
The status bar at the bottom indicates "LSP Python: ready", "conda: base (Python 3.7.9)", "Line 185, Col 1", "ASCII", "CRLF", "RW", and "Mem 86%".

IDEs for Python

```

temp.py x Par_est_problem_2.py x Par_est_problem_1-c.py x Par_est_problem_1-a.py x Trial.py x
1 #Import all the packages
2 import numpy as np
3 from scipy.integrate import odeint
4 from scipy.integrate import solve_ivp
5 import matplotlib.pyplot as plt
6 import pandas as pd
7 from scipy.optimize import curve_fit
    
```

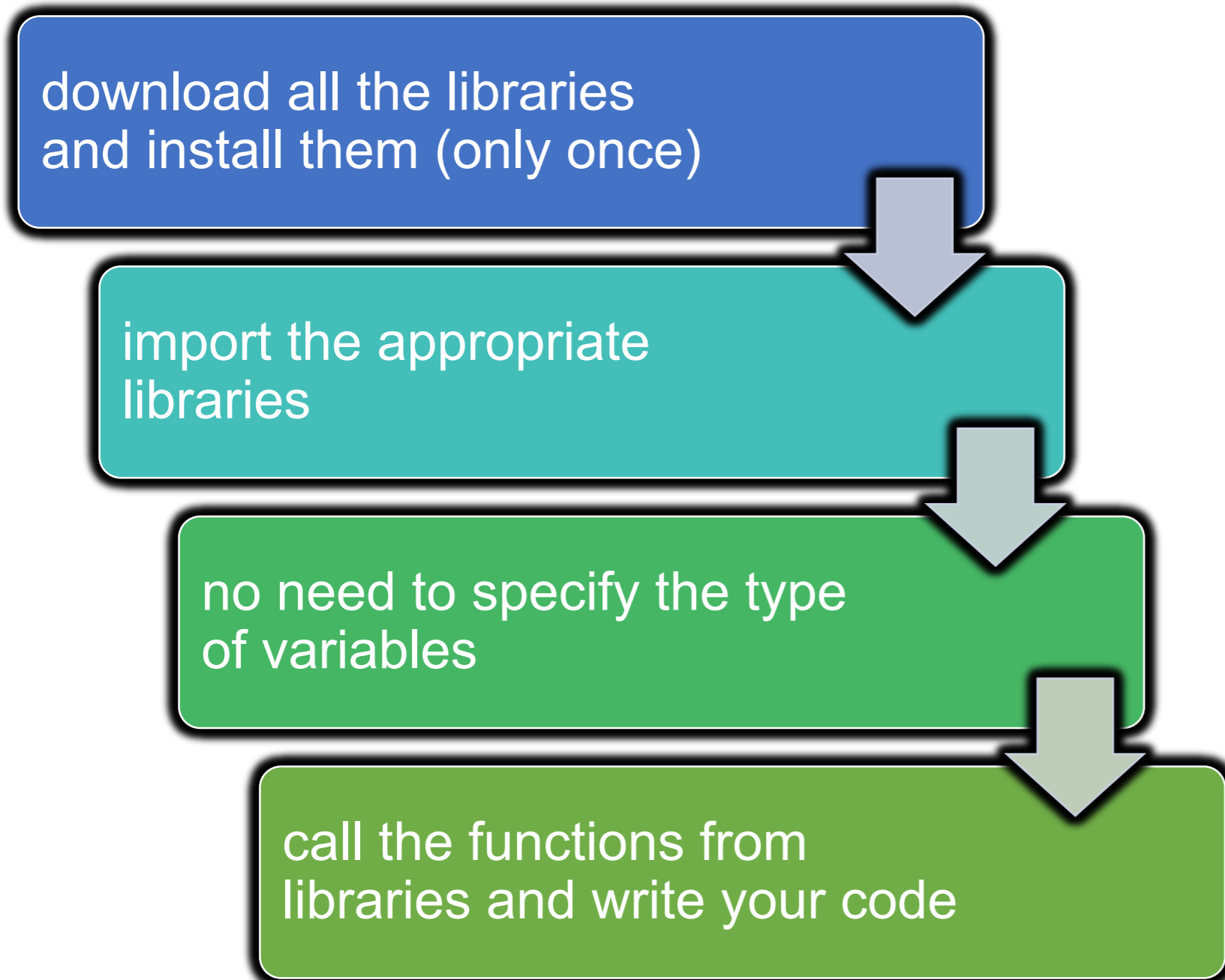
Name	Type	Size	Value
C	Array of float64	(12, 4)	[[2.6700e+00 3.7300e+00 3.9141e+03 6.3630e+02] [2.6700e+00 4.0000e+01 ...
C0	Array of float64	(12, 4)	[[2.670e+00 2.539e+01 3.918e+03 6.108e+02] [2.670e+00 5.078e+01 3.893 ...
C_comp	Array of float64	(12, 4)	[[2.67000000e+00 3.52201902e+00 3.9219399... [2.670 ...
C_model	Array of float64	(12, 4)	[[2.67000000e+00 3.52201902e+00 3.9219399... [2.670 ...
Cc	Array of float64	(12,)	[3921.93996203 3902.89971368 3874.23611914 ... 3795.02902554 3873.3647 ...
Ccurve	Array of float64	(12, 2)	[[3.730e+00 6.363e+02] [4.000e+01 6.130e+02]
color	list	4	['r*', 'bs', 'gs', 'k^']
data_file	DataFrame	(12, 9)	Column names: Experiment, Reactor length, Cb0, Cc0, Cd0, Ca, Cb, Cc, C ...
Den	Array of float64	(0,)	[]



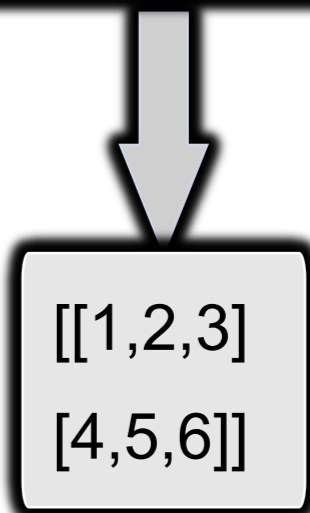
```

36 def rxn_CS(CS_sol, k1, k2, k1m, k2m, k3m, Ca, Cb, Cc):
37
38     Cas, Cbs=Cs_sol
39
40     k1=np.exp(k1)
41     k2=np.exp(k2)
42     k1m=np.exp(k1m)
43     k2m=np.exp(k2m)
    
```

Code in Python



```
#Import the numpy package. np is  
#the abbreviation for numpy.  
import numpy as np  
#Creating a matrix  
M=np.array([[1,2,3],[4,5,6]])  
print(M)
```



help function

```
F[1]=50.2*(Cb-Cbs)-r1
return F
root(fun, x0, args=(), method='hybr', jac=None, tol=None, callback=None,
      options=None)
# =====
# Guess= Find a root of a vector function.
# Cas,Cb 1.970574
# Parameters
# print(-----)
# =====
fun : callable
A vector function to find a root of.
def rxn(x0 : ndarray
Initial guess.
Ca,C args : tuple, optional
Extra arguments passed to the objective function and its Jacobian.
method : str, optional
Type of solver. Should be one of
if C
- 'hybr' :ref: `(see here) `
- 'lm' :ref: `(see here) `
- 'broyden1' :ref: `(see here) `
- 'broyden2' :ref: `(see here) `
- 'anderson' :ref: `(see here) `
- 'linear mixing' :ref: `(see here) `
- 'diagbroyden' :ref: `(see here) ` ...
# =====
# Ca
# =====
Click anywhere in this tooltip for additional help
sol=root(equations,Guess,args=(2.98412505e+02,2.52912191e+02,7.19705743e+00
Cas,Cbs=sol.x
```

SciPy.org

SciPy.org Docs SciPy v1.6.1 Reference Guide Optimization and root finding (scipy.optimize)

index modules next previous

scipy.optimize.root

`scipy.optimize.root(fun, x0, args=(), method='hybr', jac=None, tol=None, callback=None, options=None)` [\[source\]](#)

Find a root of a vector function.

Parameters:

- fun** : *callable*
A vector function to find a root of.
- x0** : *ndarray*
Initial guess.
- args** : *tuple, optional*
Extra arguments passed to the objective function and its Jacobian.
- method** : *str, optional*
Type of solver. Should be one of
 - 'hybr' ([see here](#))
 - 'lm' ([see here](#))
 - 'broyden1' ([see here](#))
 - 'broyden2' ([see here](#))
 - 'anderson' ([see here](#))
 - 'linear mixing' ([see here](#))
 - 'diagbroyden' ([see here](#))
 - 'exciting mixing' ([see here](#))
 - 'krylov' ([see here](#))
 - 'df-sane' ([see here](#))
- jac** : *bool or callable, optional*
If *jac* is a Boolean and is True, *fun* is assumed to return the value of Jacobian along with the objective function. If False, the Jacobian will be estimated numerically. *jac* can also be a callable returning the Jacobian of *fun*. In this case, it must accept the same arguments as *fun*.

Previous topic: [scipy.optimize.fixed_point](#)

Next topic: [root\(method='hybr'\)](#)

Quick search:

help function

input/output of function

```
# Cas,Cb
```

```
[970574
```

default settings

```
Extra arguments passed to the objective function and its Jacobian.
```

methods

```
# ===== - 'linearmixing' :ref: `(see here)`  
# Ca - 'diagbroyden' :ref: `(see here)` ...  
# ===== Click anywhere in this tooltip for additional help  
sol=root(equations,Guess,args=(2.98412505e+02,2.52912191e+02,/.19705743e+00  
Cas,Cbs=sol.x
```

method : *str, optional*

Type of solver. Should be one of

- 'hybr' (see here)
- 'lm' (see here)
- 'broyden1' (see here)
- 'broyden2' (see here)
- 'anderson' (see here)
- 'linearmixing' (see here)
- 'diagbroyden' (see here)
- 'excitingmixing' (see here)
- 'krylov' (see here)
- 'df-sane' (see here)

help function

input/output of function

default settings

methods

example

Examples

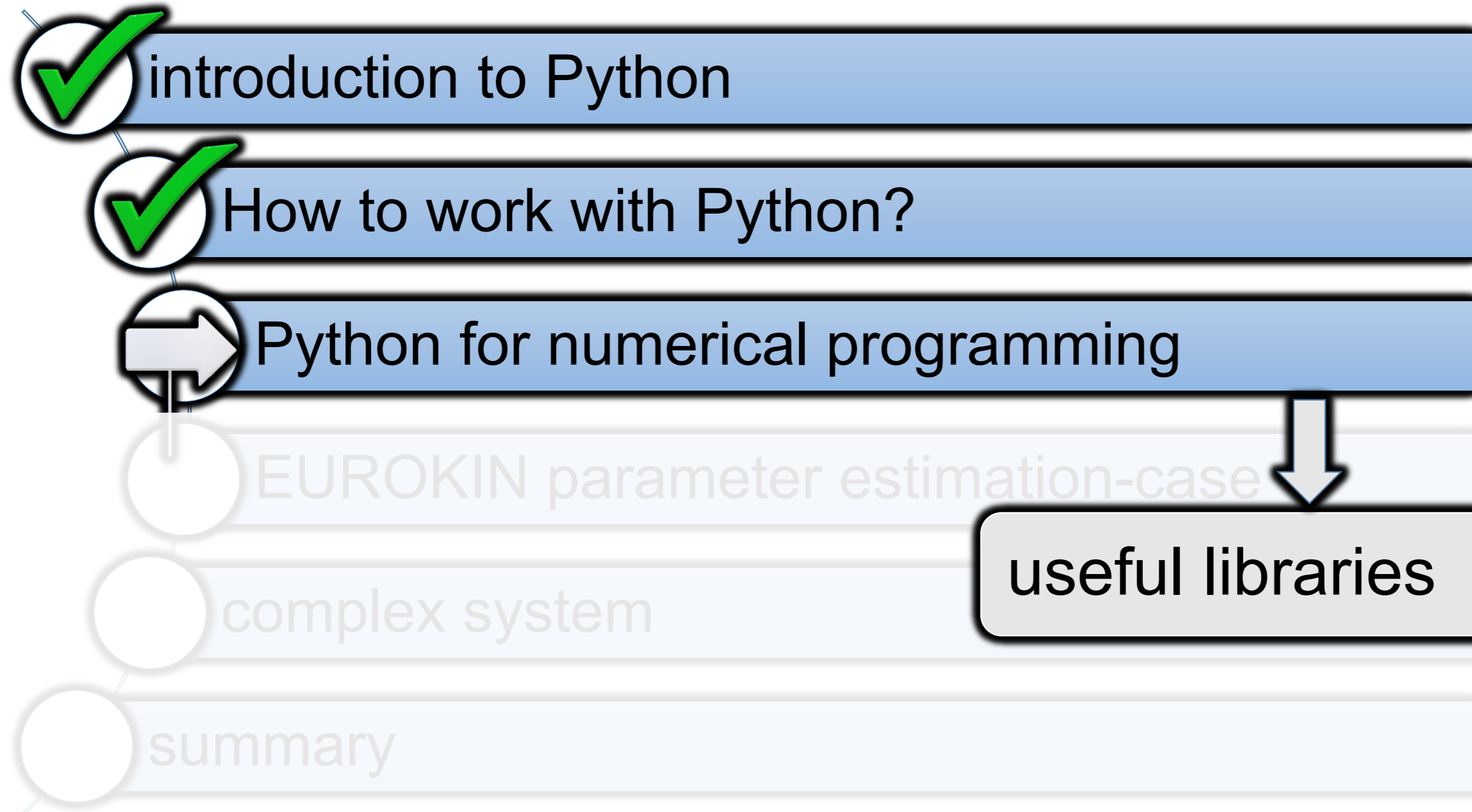
The following functions define a system of nonlinear equations and its jacobian.

```
>>> def fun(x):
...     return [x[0] + 0.5 * (x[0] - x[1])**3 - 1.0,
...             0.5 * (x[1] - x[0])**3 + x[1]]
>>> def jac(x):
...     return np.array([[1 + 1.5 * (x[0] - x[1])**2,
...                       -1.5 * (x[0] - x[1])**2],
...                      [-1.5 * (x[1] - x[0])**2,
...                       1 + 1.5 * (x[1] - x[0])**2]])
```

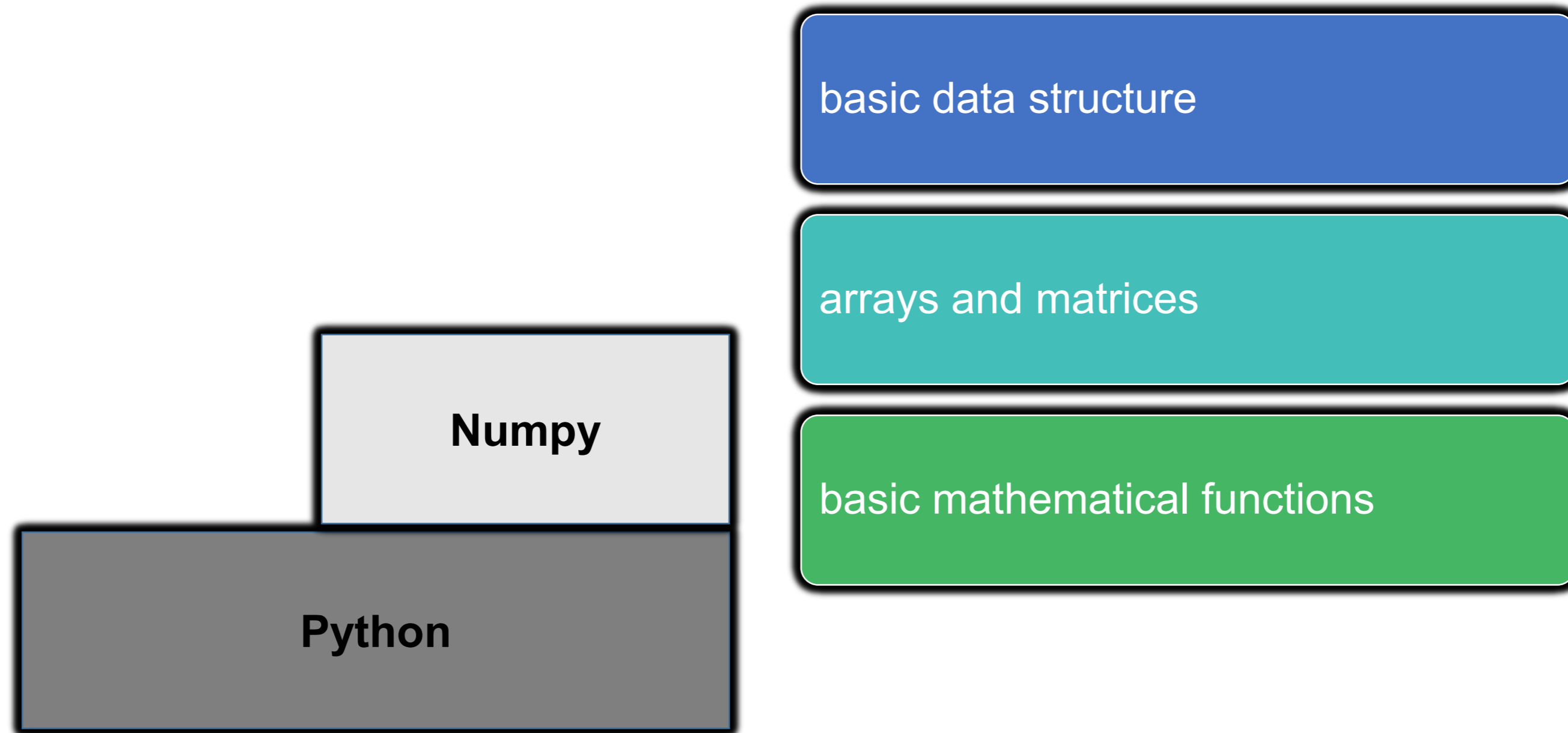
A solution can be obtained as follows.

```
>>> from scipy import optimize
>>> sol = optimize.root(fun, [0, 0], jac=jac, method='hybr')
>>> sol.x
array([ 0.8411639,  0.1588361])
```

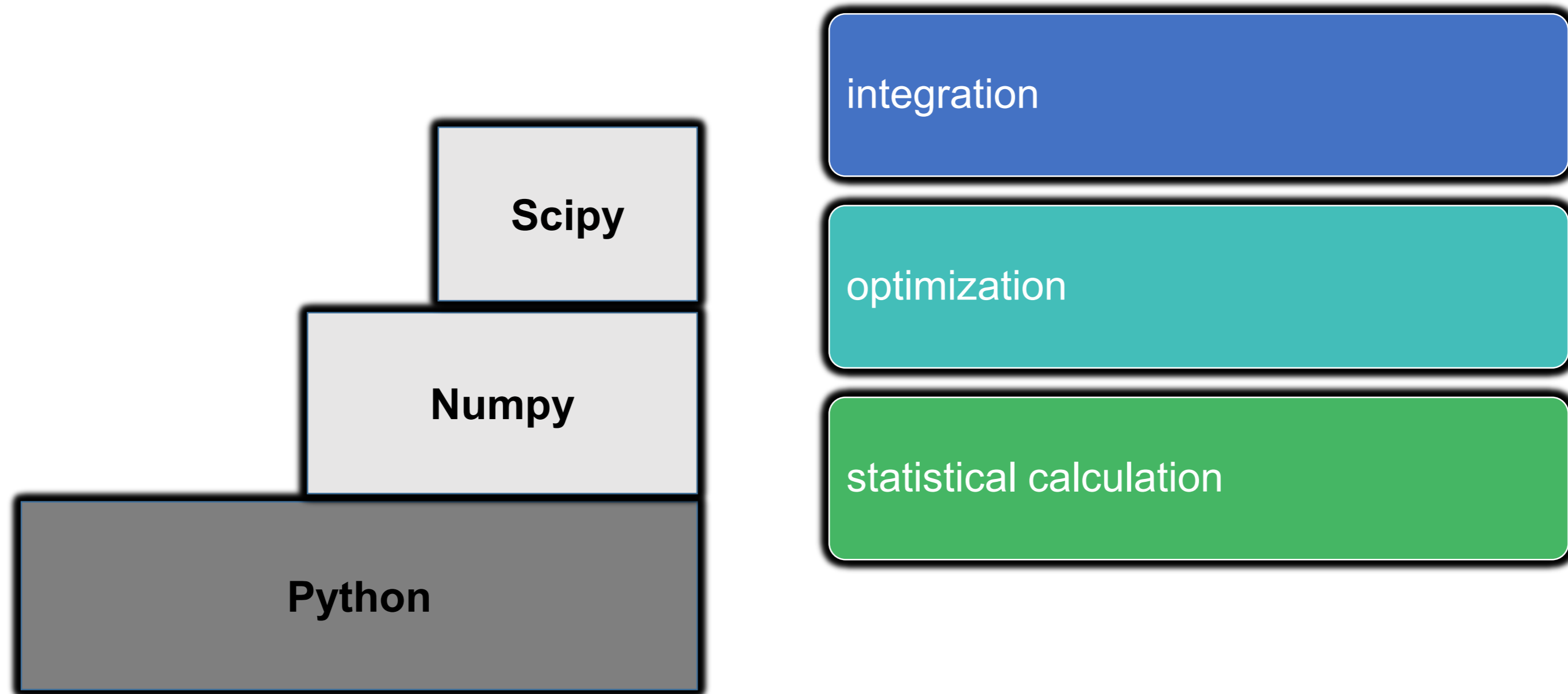
overview



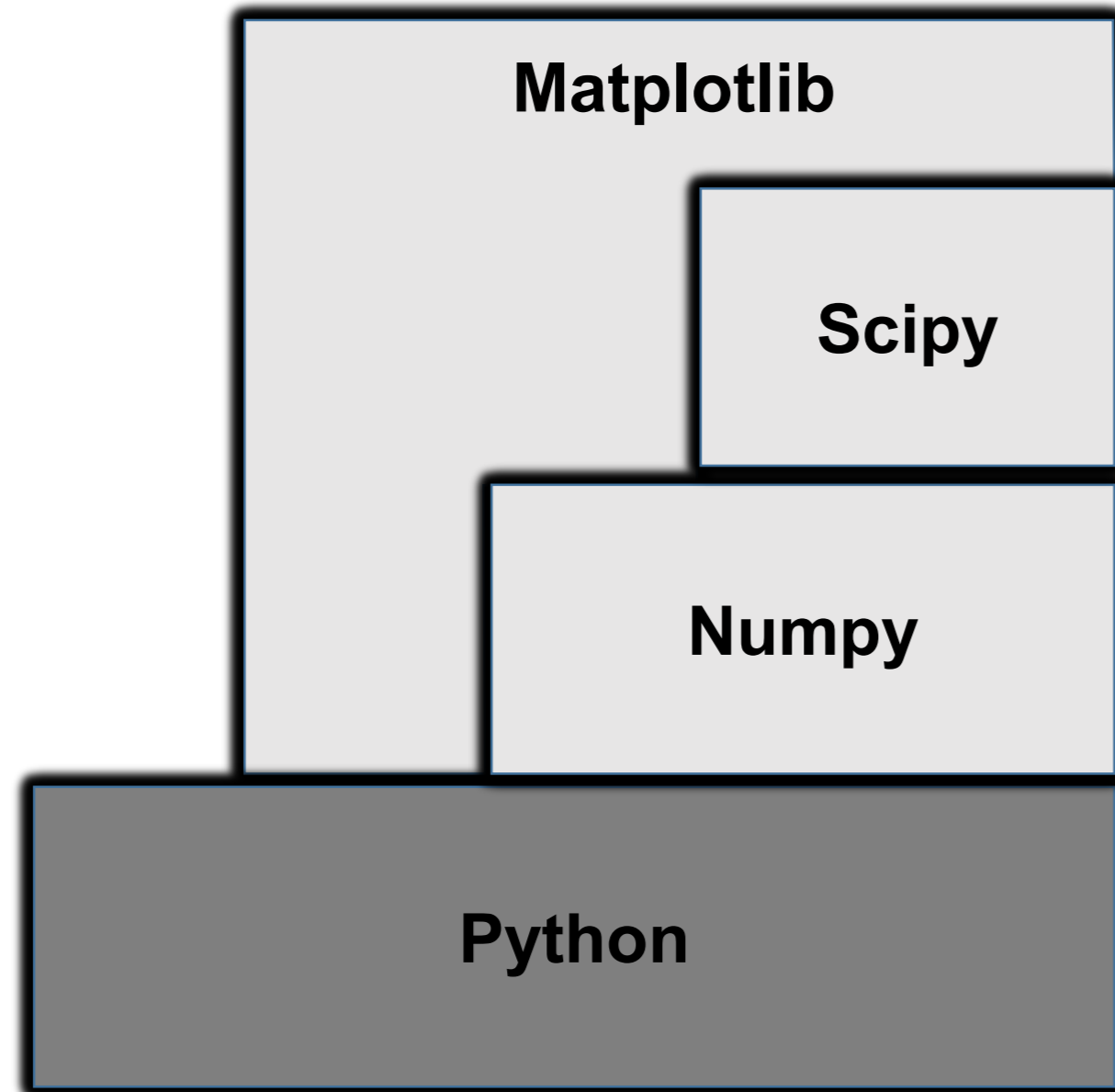
Python for numerical programming



Python for numerical programming

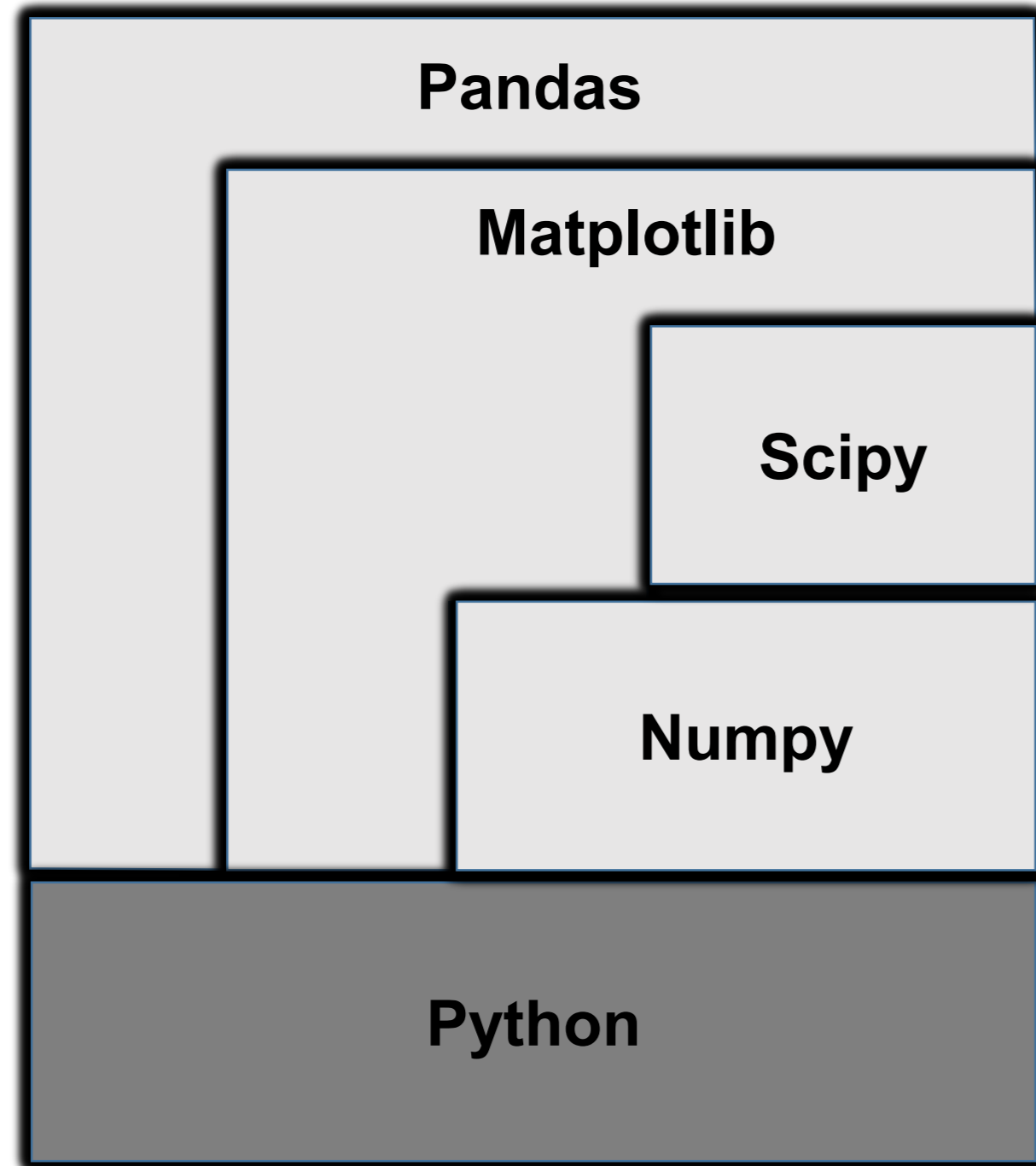


Python for numerical programming



Data visualization

Python for numerical programming



Data structure

Import/export data

overview

✓ introduction to Python

✓ How to work with Python?

✓ Python for numerical programming

➔ EUROKIN parameter estimation-case 1

Complex system

Summary

problem statement

Solution procedure for different scenarios:

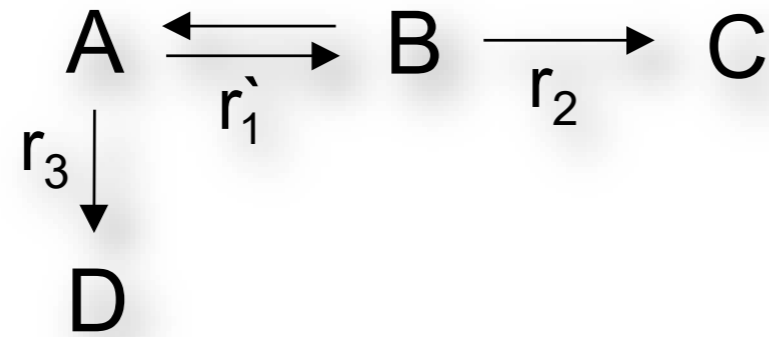
Isothermal batch reactor data

Non-isothermally assessment of batch reactor data

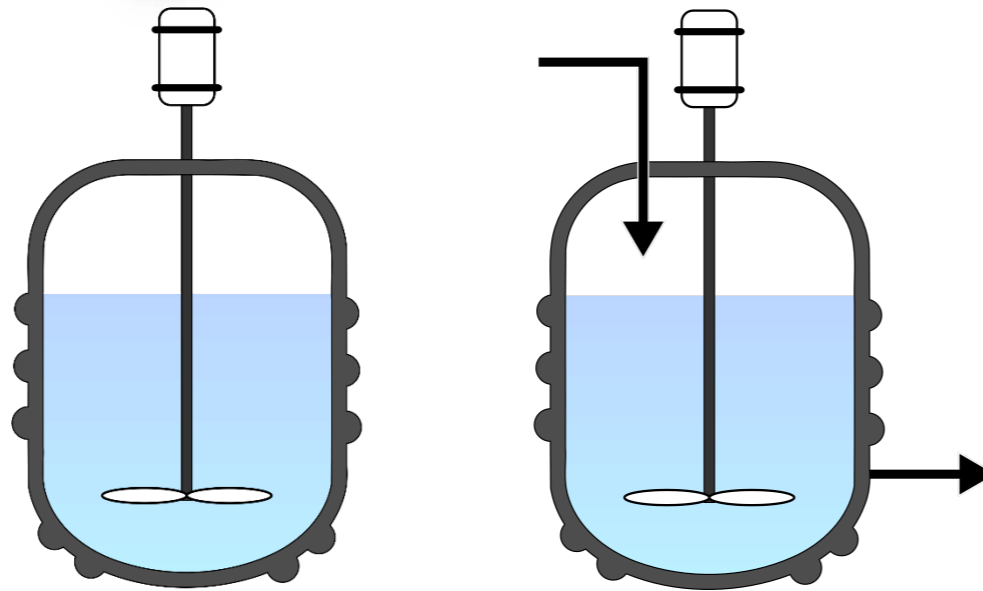
CSTR and batch reactor data

problem statement

reaction?



reactors?

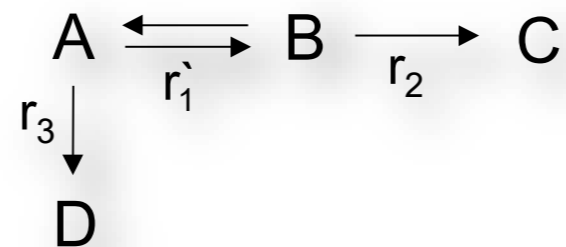
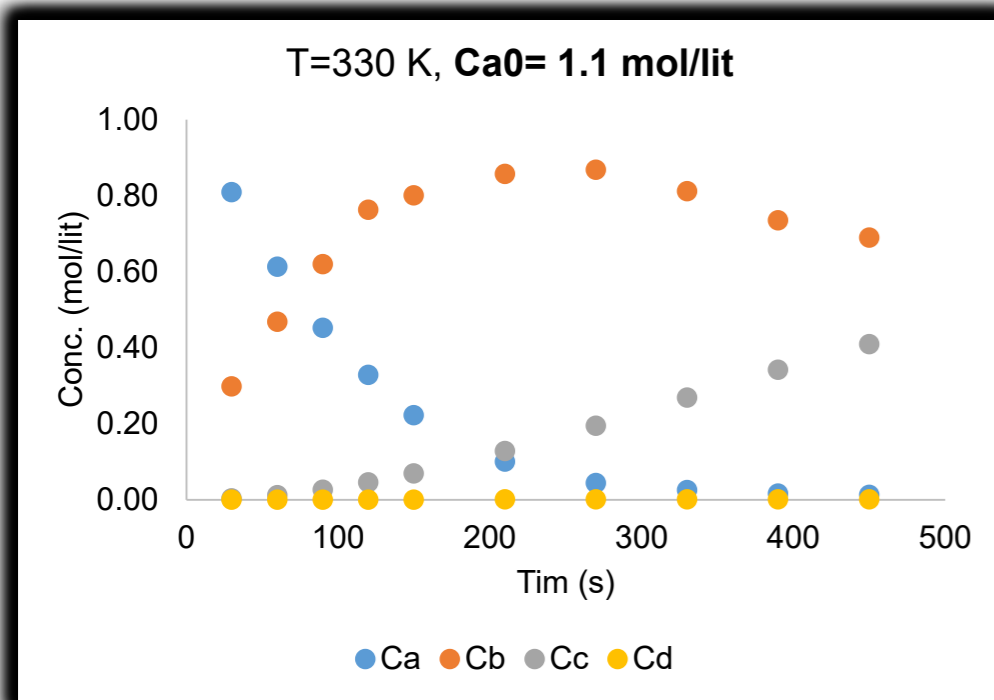
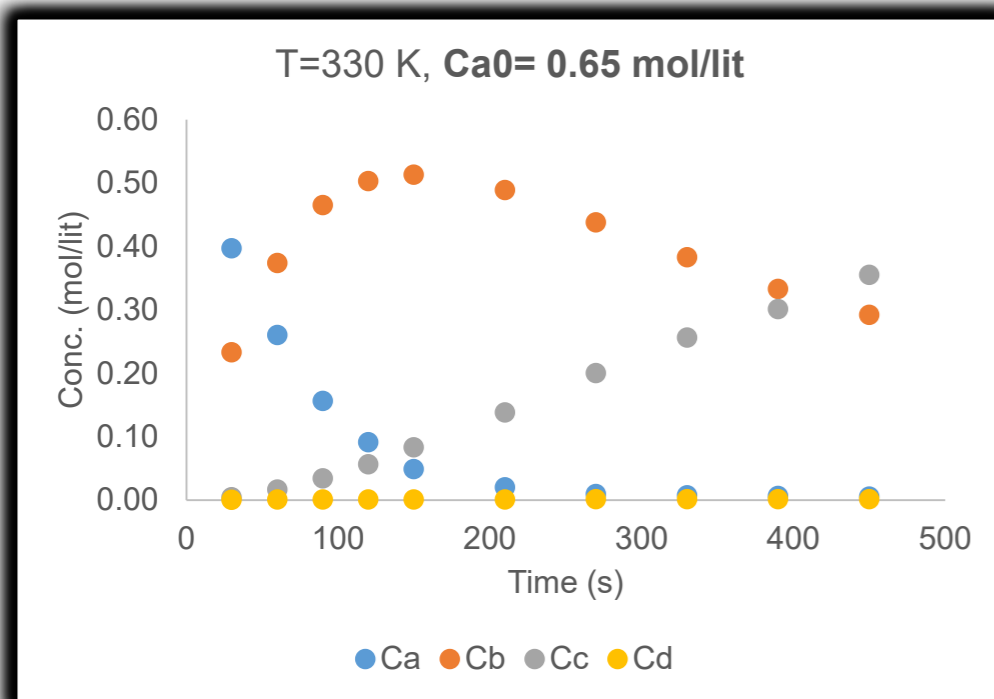


temperature?

310 K 330 K 360 K

	part 1	part 2	part 3
batch reactor?	✓	✓	✗
CSTR?	✗	✗	✗
isothermal data?	✓	✗	✗
non-isothermal data?	✗	✗	✗

part1: batch reactor + isothermal data

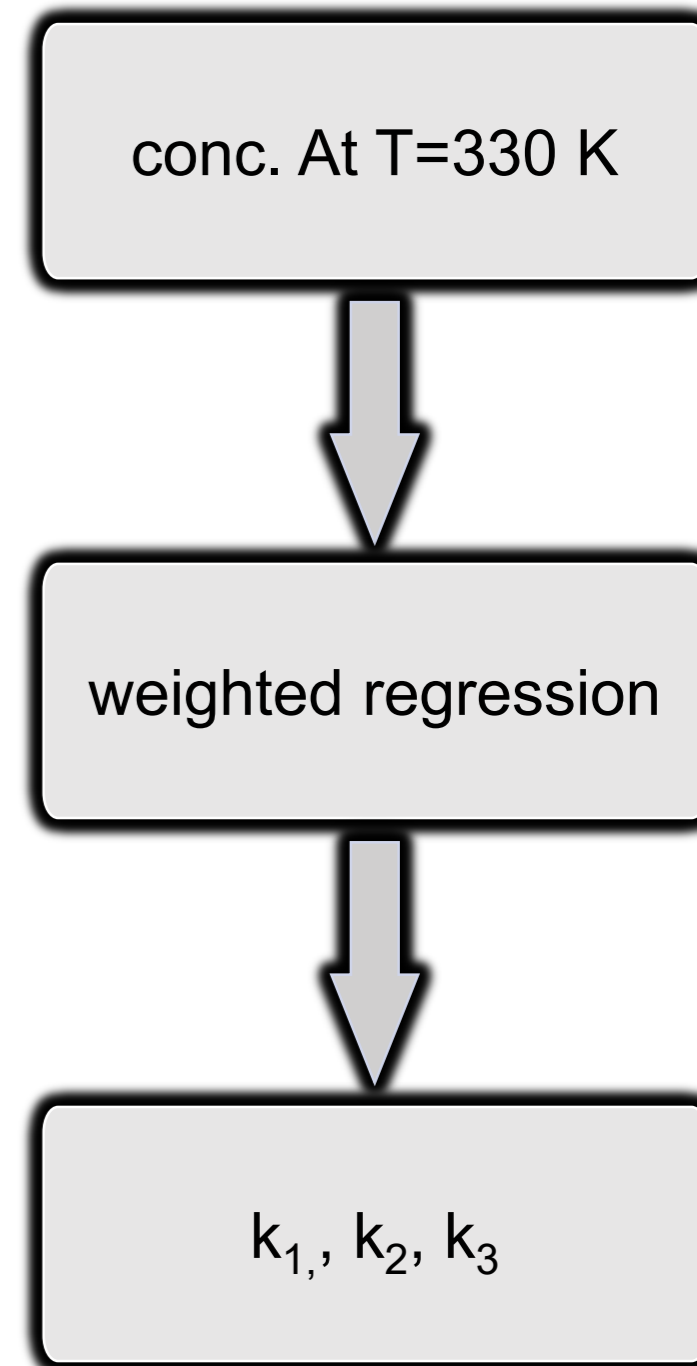


$$\frac{dC_A}{dt} = \frac{-k_1 K_A \left(C_A - \frac{C_B}{K_{eq1}} \right) - k_3 K_A C_A}{1 + K_A C_A + K_B C_B}$$

$$\frac{dC_C}{dt} = \frac{k_2 K_B C_B}{1 + K_A C_A + K_B C_B}$$

$$\frac{dC_D}{dt} = \frac{k_3 K_A C_A}{1 + K_A C_A + K_B C_B}$$

$$C_B = C_{A0} - C_A - C_C - C_D$$



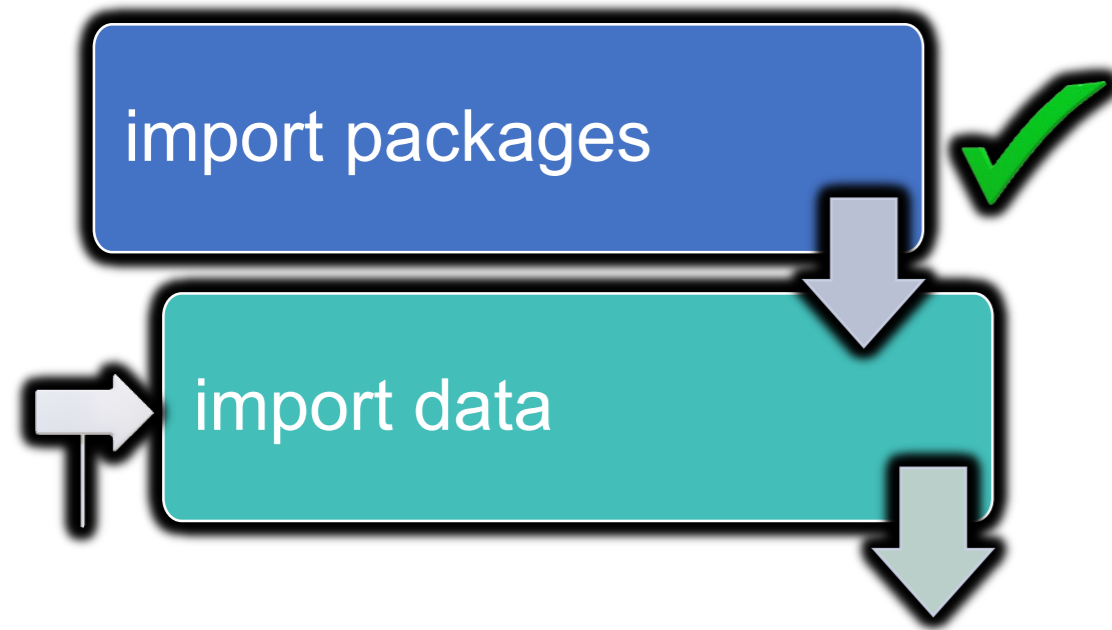
part1: batch reactor + isothermal data (code)



import packages

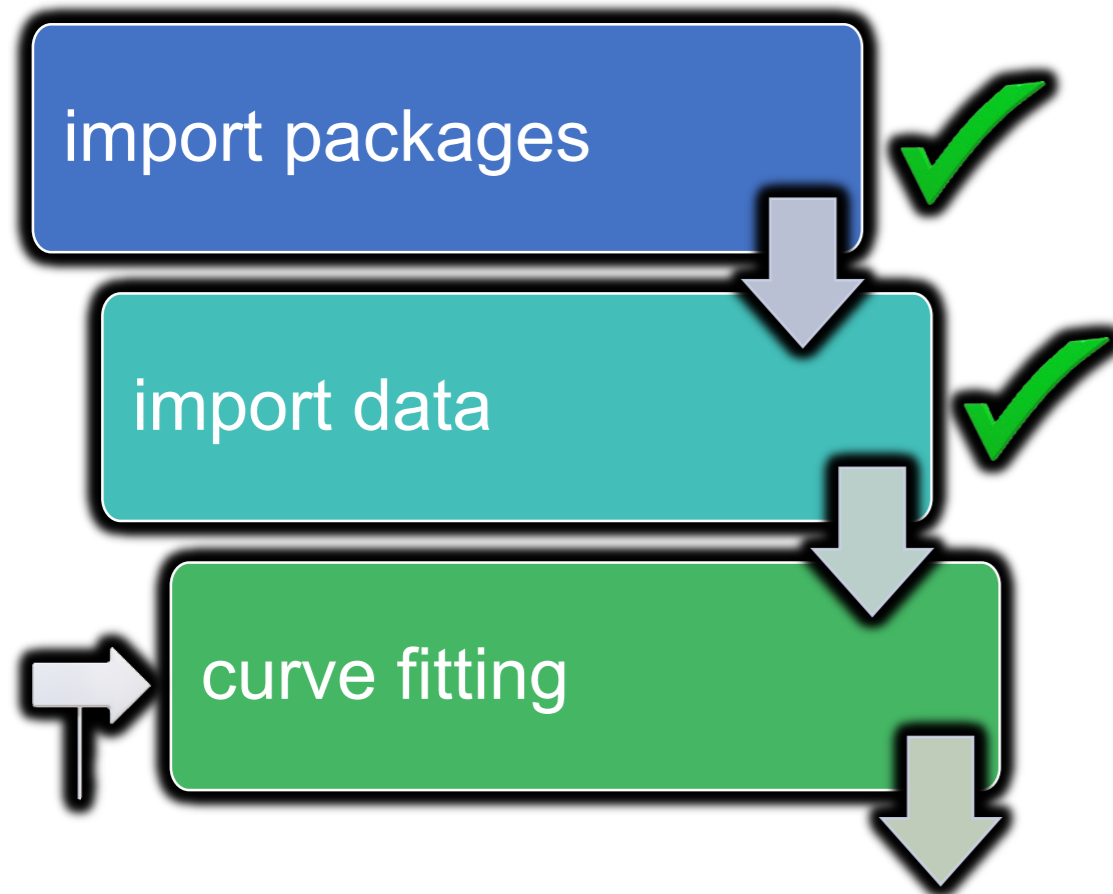
```
1 #Import all the packages
2 import numpy as np
3 import pandas as pd
4 from scipy.integrate import odeint
5 from scipy.optimize import curve_fit
6 from scipy.stats import probplot
7 from scipy.stats import f
8 from scipy.stats import t as t_test
9 import matplotlib.pyplot as plt
10 #This package compute confidence intervals
11 from uncertainties import ufloat
```

part1: batch reactor + isothermal data (code)

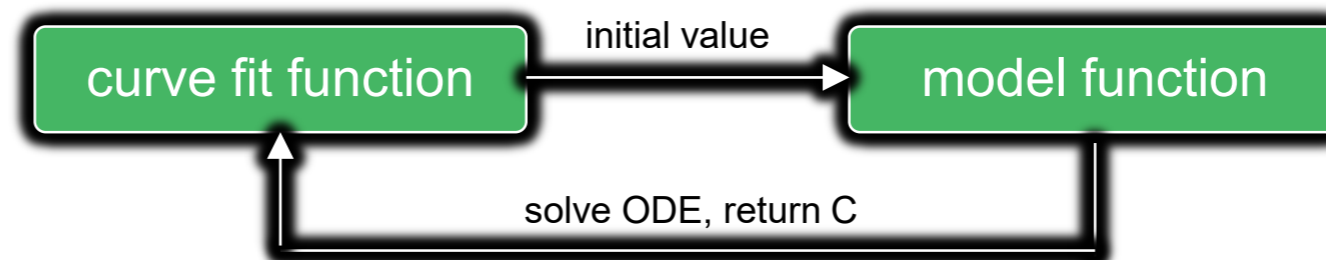


```
16 #Read data from excel
17 data_file = pd.read_excel('Data.xlsx')
18
19 t=data_file['Time']
20 data_file1=data_file.loc[:, 'Ca1': 'Cd2']
21 C_data=pd.DataFrame(data_file1).to_numpy()
```

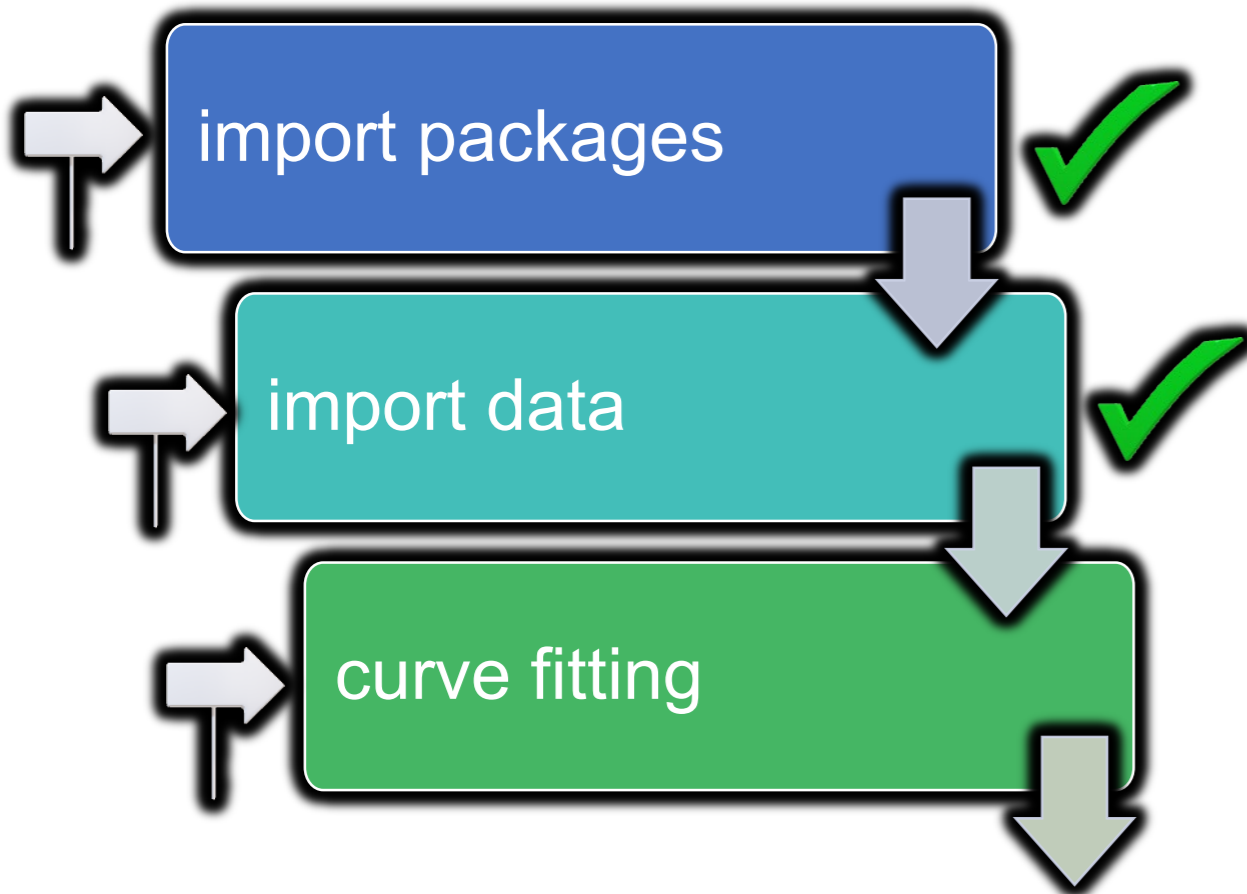
part1: batch reactor + isothermal data (code)



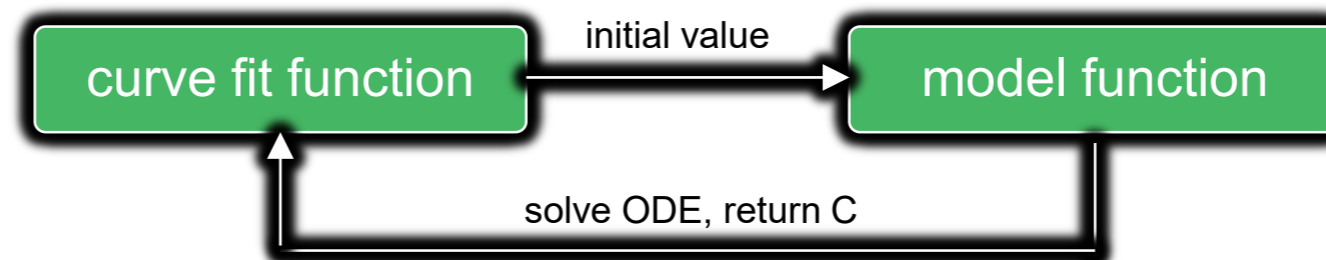
```
84 #Define heteroscedasticity factor
85 n0=2
86 nf=2
87 n=np.arange(n0,nf+0.0000001,0.1)
88
89 #Define zero matrices for saving the parameters calculated in the for loop
90 k1=np.zeros(len(n))
91 k2=np.zeros(len(n))
92 k3=np.zeros(len(n))
93 R2=np.zeros(len(n))
94
95 #Define upper and lower limit bound for parameters
96 lowb=np.array([0,0,0])
97 upb=np.array([0.1,0.1,0.001])
98
99 #This for loop calculate for each n value
100 for i in range(len(n)):
101     print('\nn= '+str('{:.3f}'.format(n[i])))
102     #Define weights for curve_fit function
103     weight=(C_data1.ravel(order='F'))**(n[i]/2)
104
105     #define weight for determining weighte residuals
106     w=(C_data1)**(n[i]/2)
107
108     #Optimize parameters using weights. The curve fit function returns
109     #optimized values and covariance matrix.
110     popt,pcov=curve_fit(model,t,C_data1.ravel(order='F'),\
111         p0=[0.01,0.001,0.00001],bounds=(lowb,upb),sigma=weight)
112
```



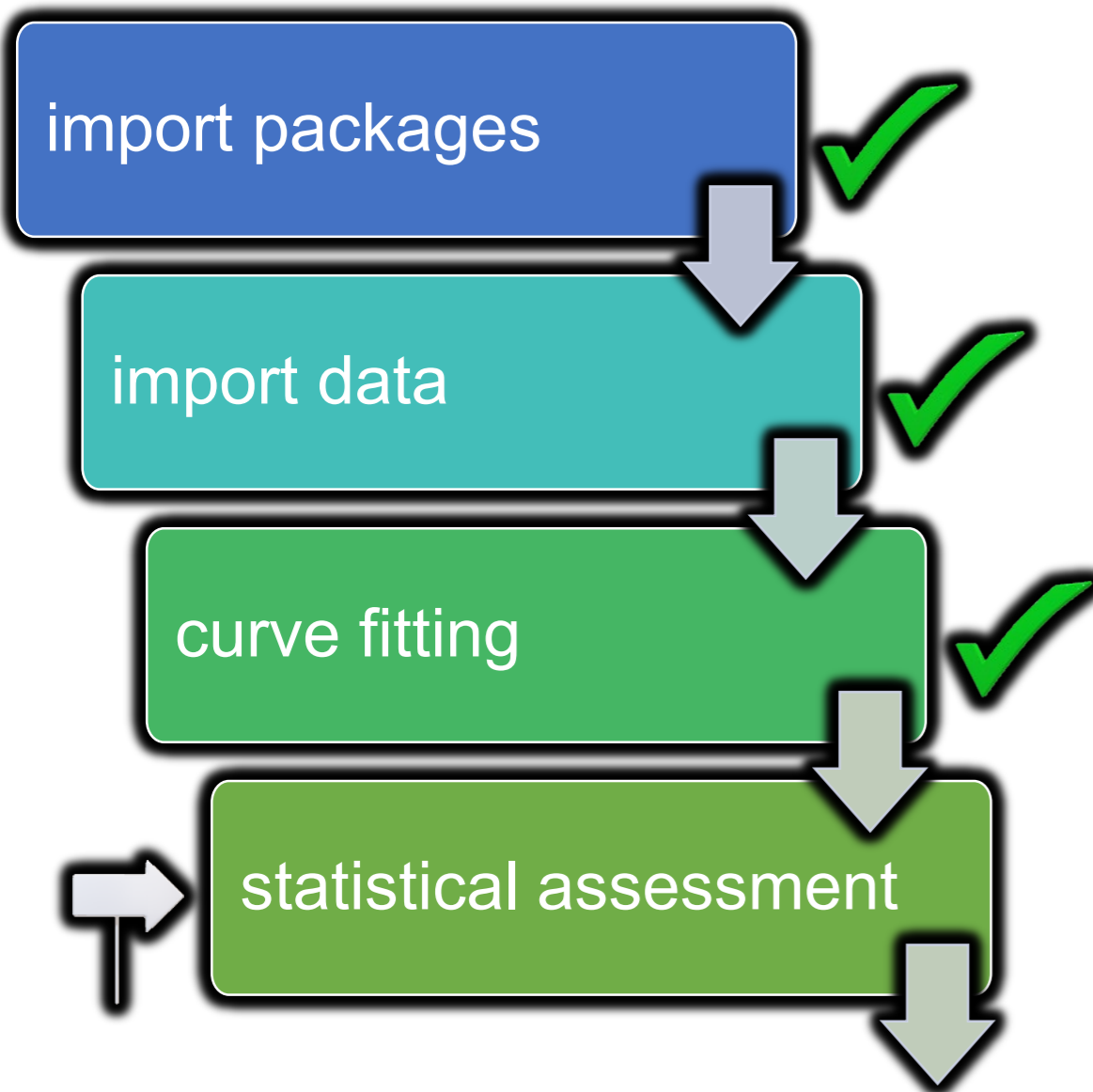
part1: batch reactor + isothermal data (code)



```
49 def model(t,k1,k2,k3):
50
51     #add t=0 to the Time vector-because the ODEint needs the initial values.
52     tt=np.array([0])
53     tt=np.append(tt,t)
54     #Initial concentration of components for two batches
55     C0=np.array([[0.65,0,0,0],[1.1,0,0,0]])
56     #Creating zero matrices for Cb and solved cocentrations. +1 because we
57     #have added t=0 to the time vector. so we need to add a row for that.
58     C_sol=np.zeros((len(t)+1,np.size(C0)))
59     Cb=np.zeros((len(t)+1,len(C0)))
60
61     #-----
62     #This for loop solve the ODEs, save them in C_sol matrix. since the Cb has
63     #to be calculated by material balance, the value calculated by ode is
64     #replaced to material balance value.
65     for i in range(len(C0)):
66
67         C_sol[:,i*4:i*4+4]=odeint(rxn,C0[i,:],tt,args=(k1,k2,k3))
68         Cb[:,i]=C0[i,0]-C_sol[:,i*4]-C_sol[:,i*4+2]-C_sol[:,i*4+3]
69         C_sol[:,i*4:i*4+4]=np.c_[C_sol[:,i*4],Cb[:,i],C_sol[:,i*4+2],\
70                                 C_sol[:,i*4+3]]
71     #-----
72     #remove the initial concentrations and then send them back.
73     C_sol=C_sol[1:,:]
74     #Returns the flattened concentration to Curve_fit function.
75     return C_sol.ravel(order='F')
```



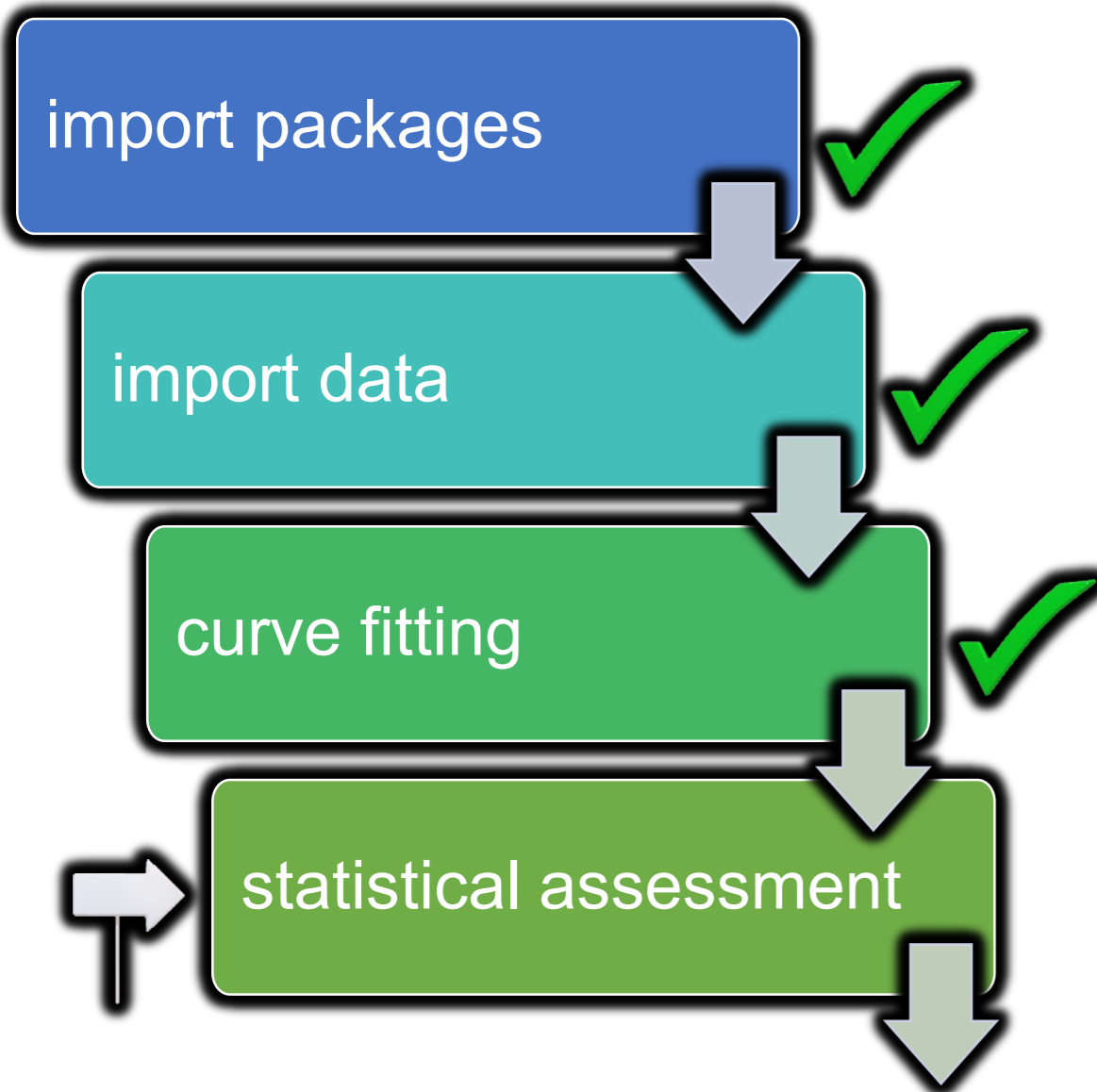
part1: batch reactor + isothermal data (procedure)



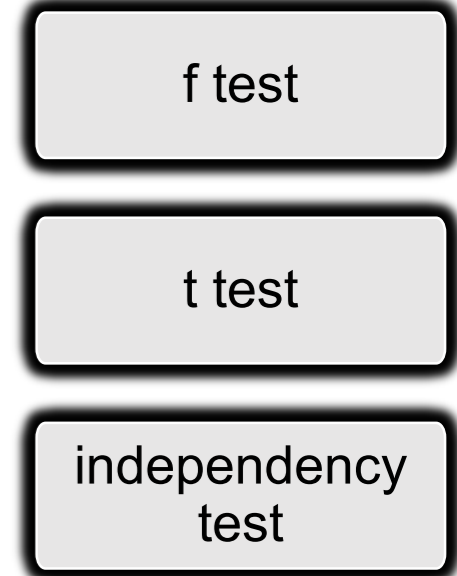
```
122 #Determine F value
123 if i==0:
124     Fs_cal=np.zeros(len(n))
125     Res=((C_data[:, :]-C_comp[:, :])/w[:, :])
126     SSQreg=(C_comp[:, :]**2).sum()
127     SSQres=((C_data[:, :]-C_comp[:, :])**2).sum()
128
129     #degrees of freedom for numerator and denominator
130     df1=len(pcov)
131     df2=(len(C_data))*np.size(C_data,1)-df1
132
133     Fs_cal[i]=(SSQreg/df1)/(SSQres/df2)
134
135     Fs_tab = f.ppf(q=1-0.05, dfn=df1, dfd=df2)
136
137     #Check the signficancy of the model
138     print('-F test')
139
140     if Fs_cal[i]>Fs_tab:
141         print('The model is significant')
142     else:
143         print('The model is NOT significant')
144
```

f test

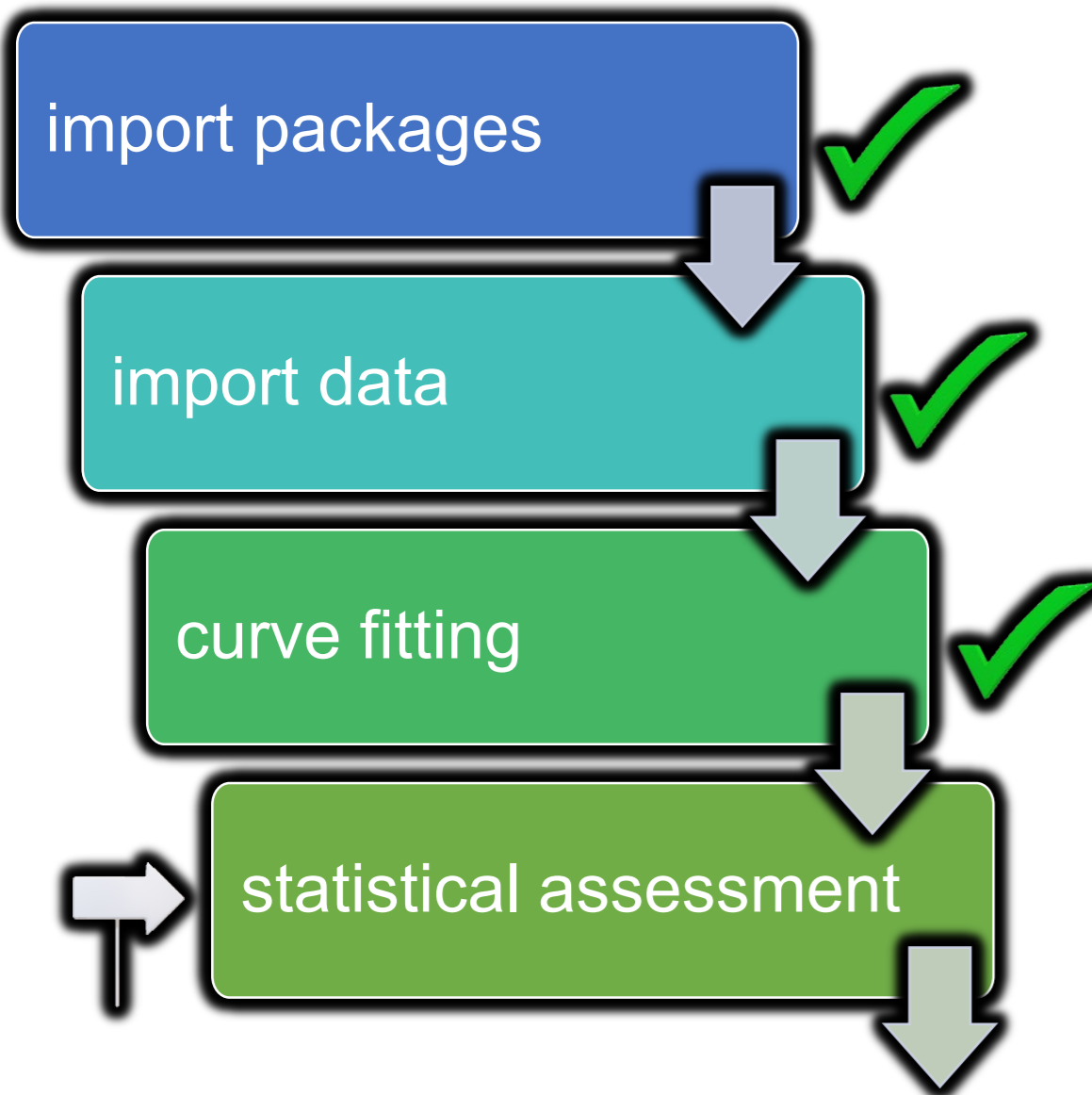
part1: batch reactor + isothermal data (procedure)



```
146 #t-test
147 if i==0:
148     t_cal=np.zeros((len(n),len(pcov)))
149
150 t_tab = t_test.ppf((1+0.95)/2, df2)
151
152 print('\n-t test (only paratemetrs that failed the test are shown)')
153 for j in range(len(pcov)):
154
155     t_cal[i,j]=K[i,j]/(pcov[j,j])**0.5
156
157     if t_cal[i,j]<t_tab:
158         print('    k'+str(j+1)+' is not significantly different from zero')
159 # =====
160 #     elif :
161 #         print('k'+str(j+1)+' is significantly different from zero')
162 # =====
163 #ru test (correlation coefficient matrix)
164 ru=np.zeros([len(pcov),len(pcov)])
165 print('\n-Parameters independency (only strongly correlated'+\
166     ' parameters are displayed)')
167 for j in range(len(pcov)):
168     for k in range(len(pcov)):
169
170         ru[j,k]=pcov[j,k]/(pcov[j,j]*pcov[k,k])**0.5
171
172         if abs(ru[j,k])>0.95 and k>j:
173             print('    The parameters k' + str(j+1)+ ' and k'+str(k+1)+\
174                 ' are strongly correlated')
```



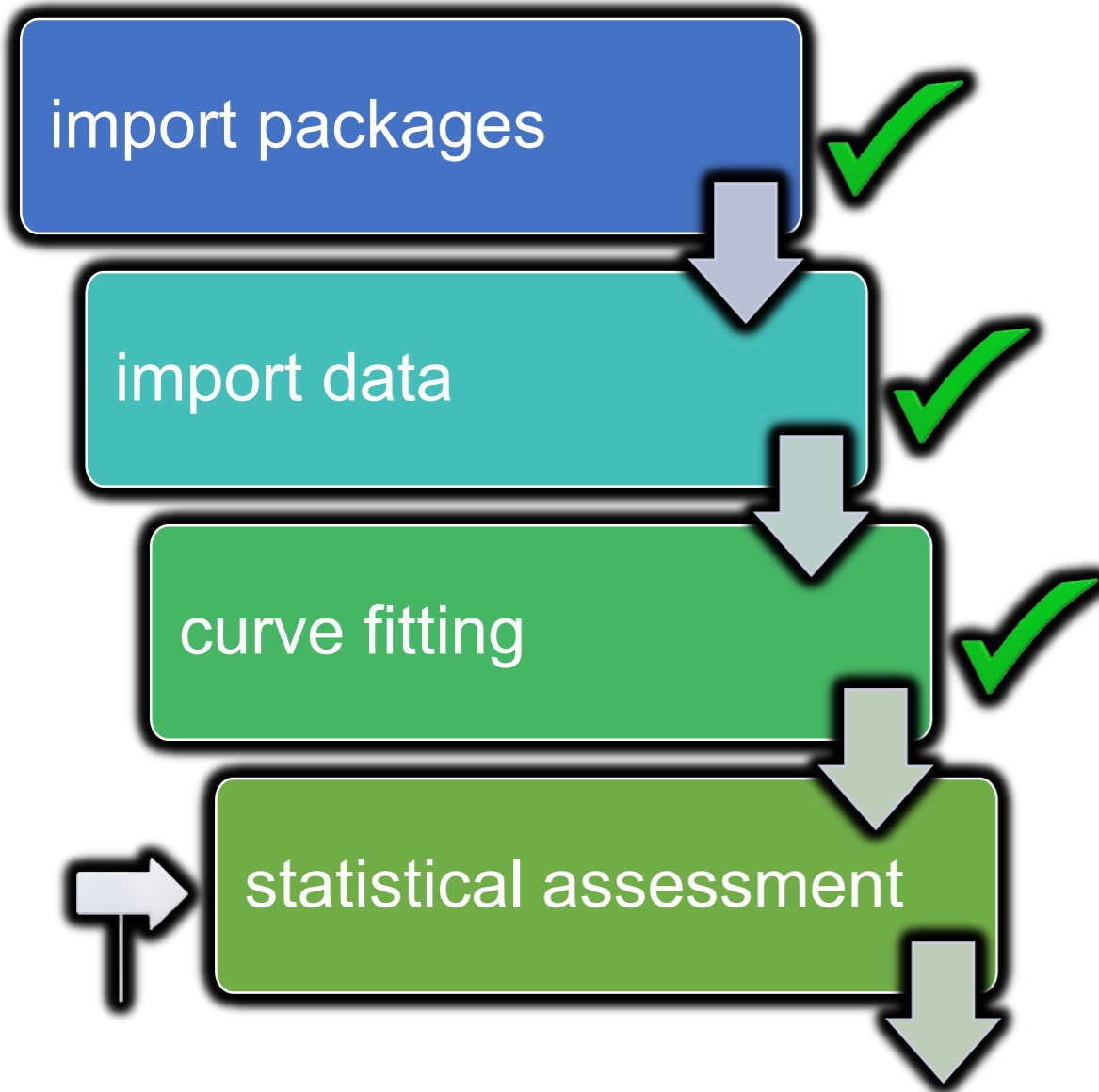
part1: batch reactor + isothermal data (procedure)



```
182 #Parity diagram
183 ABCD=['A', 'B', 'C', 'D']
184 color=['r*', 'bs', 'gs', 'k^']
185 for l in range(int(np.size(C_data,1)/2)):
186
187     plt.plot(C_comp[:,[1,l+4]].ravel(order='F'),C_data[:,[1,l+4]].\
188             ravel(order='F'),color[l])
189     plt.plot(C_comp[:,[1,l+4]].ravel(order='F'),C_comp[:,[1,l+4]].\
190             ravel(order='F'),'k-')
191     plt.title('Parity Diagram Component '+ABCD[l]+'      n='+\
192             str('{:.3f}'.format(n[i])))
193     plt.xlabel('Computed Concentration (mol/lit)')
194     plt.ylabel('Experimental Concentration (mol/lit)')
195     plt.savefig('Parity-IB- '+ABCD[l]+'.svg')
196     plt.show()
197
198 #Weighted residual diagram
199 for l in range(int(np.size(C_data,1)/2)):
200
201     plt.plot(C_comp[:,[1,l+4]].ravel(order='F'),Res[:,[1,l+4]].\
202             ravel(order='F'),'r*')
203     plt.plot(C_comp[:,[1,l+4]],C_comp[:,[1,l+4]]*0,'k-')
204     plt.xlabel('Computed Concentration(mol/lit)')
205     plt.ylabel('Weighted Residual')
206     plt.title('Residual Figure Component '+ABCD[l]+'      n='+\
207             str('{:.3f}'.format(n[i])))
208     plt.savefig('Residual-IB '+ABCD[l]+'.svg')
209     plt.show()
210
```

- f test
- t test
- independency test
- parity diagram
- residual diagram

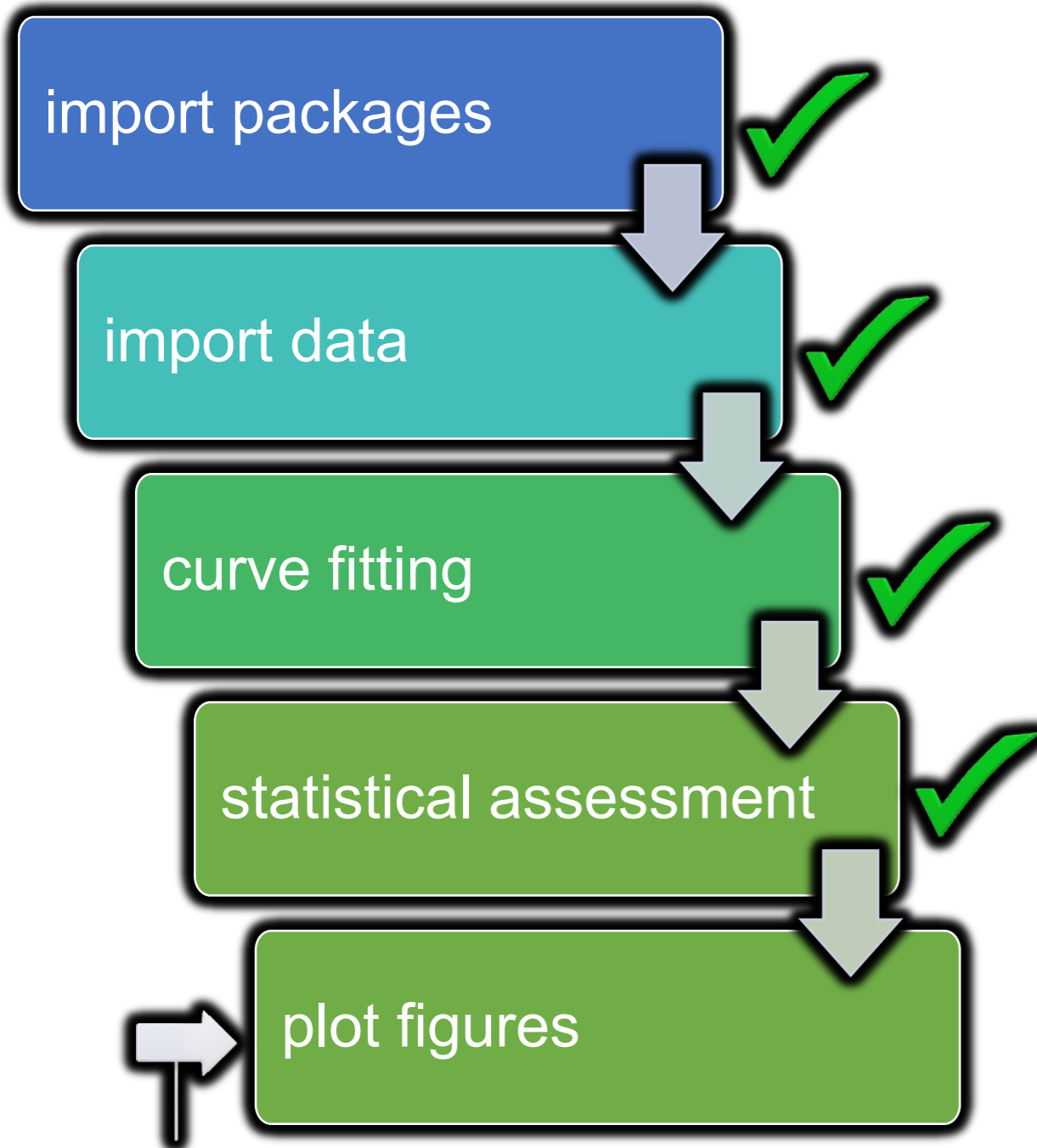
part1: batch reactor + isothermal data (procedure)



```
211 #Normal probability diagram
212 for l in range(int(np.size(C_data,1)/2)):
213
214     (OSR,fiting)=probplot(Res[:,[1,l+4]].ravel(order='F'),\
215         dist="norm", plot = plt,fit=True,rvalue=True)
216     plt.title('Probability Plot Component '+ABCD[l]+\
217         ', n='+str('{:.3f}'.format(n[i])))
218     plt.savefig('probability plot-IB '+ABCD[l]+' .svg')
219     plt.show()
```

- f test
- t test
- independency test
- parity diagram
- residual diagram
- normal probability plot

part1: batch reactor + isothermal data (procedure)

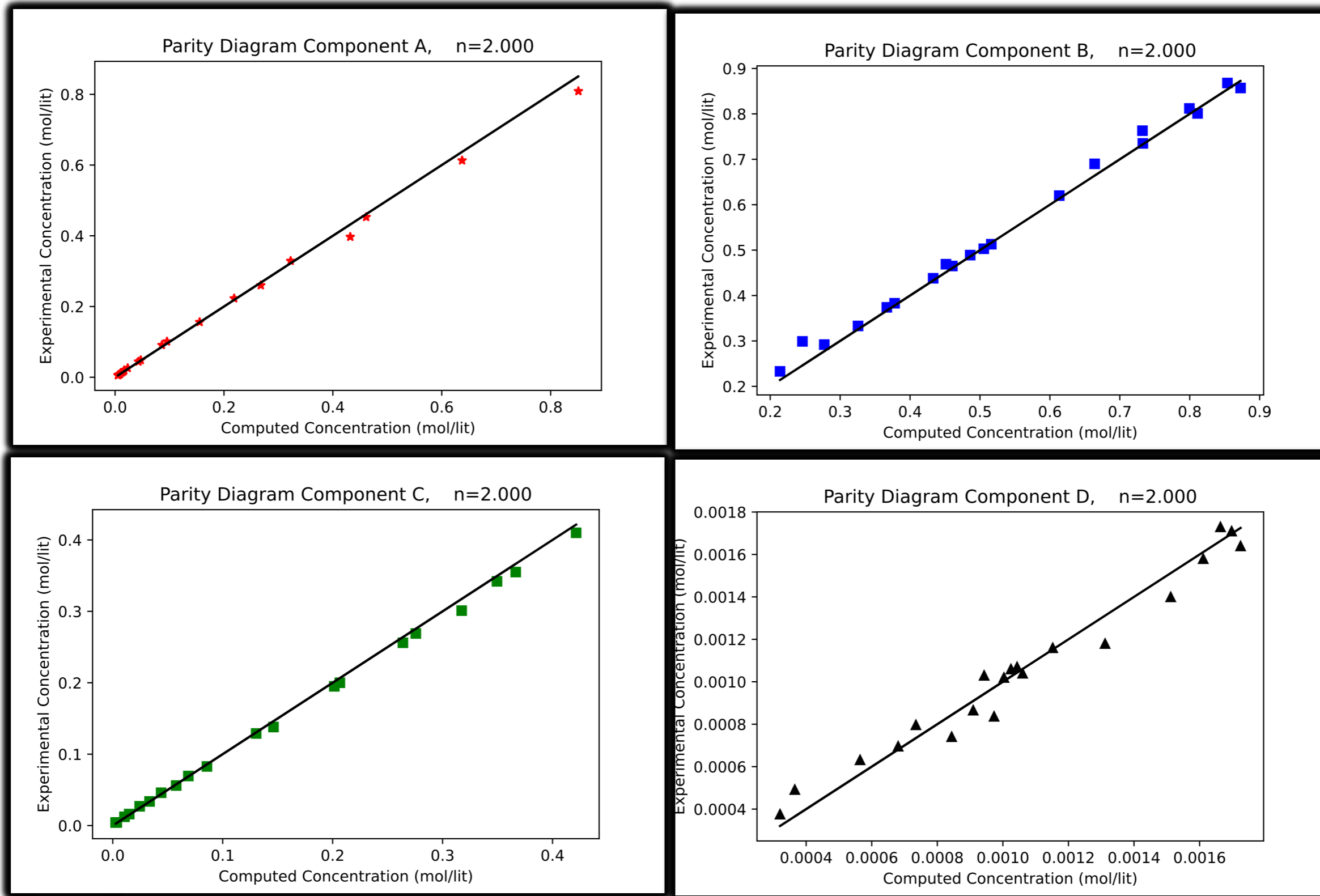


```
282 t1=np.arange(t0,tf+0.01,3)
283 C_model=model(t1,K[j,0],K[j,1],K[j,2]).reshape(8,len(t1)).T
284
285 #Generate Concentration vs. time plots
286 for i in range(int(np.size(C_data,1)/4)):
287
288     plt.figure(i)
289     plt.plot(t,C_data[:,i*4], 'r*', label='Ca'+str(i+1)+ ' exp')
290     plt.plot(t,C_data[:,i*4+1], 'bs', label='Cb'+str(i+1)+ ' exp')
291     plt.plot(t,C_data[:,i*4+2], 'g+', label='Cc'+str(i+1)+ ' exp')
292     plt.plot(t,C_data[:,i*4+3], 'k^', label='Cd'+str(i+1)+ ' exp')
293
294     plt.plot(t1,C_model[:,i*4], 'r--')#, label='Ca'+str(i+1)+ ' opt')
295     plt.plot(t1,C_model[:,i*4+1], 'b-')#, label='Cb'+str(i+1)+ ' opt')
296     plt.plot(t1,C_model[:,i*4+2], 'g:')#, label='Cc'+str(i+1)+ ' opt')
297     plt.plot(t1,C_model[:,i*4+3], 'k-')#, label='Cd'+str(i+1)+ ' opt')
298
299     plt.title('Concentration vs. time- batch')
300 # =====
301 #     plt.text(0.5,0.8,text)
302 # =====
303     plt.xlabel('Time (s)')
304     plt.ylabel('Concentration (mol/lit)')
305     plt.legend(ncol=2)
306     plt.savefig('results'+str(i+1)+'.svg')
307
```

part1: batch reactor + isothermal data (result)

parameter/diagram	outputs		
k values	$k_1=0.01016$	$k_2=0.00195$	$k_3=1.49 \cdot 10^{-5}$
F test	$2.1 \cdot 10^4 > 2.72$		✓
T test	$91 > 2$ ✓	$50 > 2$ ✓	$50 > 2$ ✓
Independent parameters?	✓	✓	✓
parity diagram	✓		

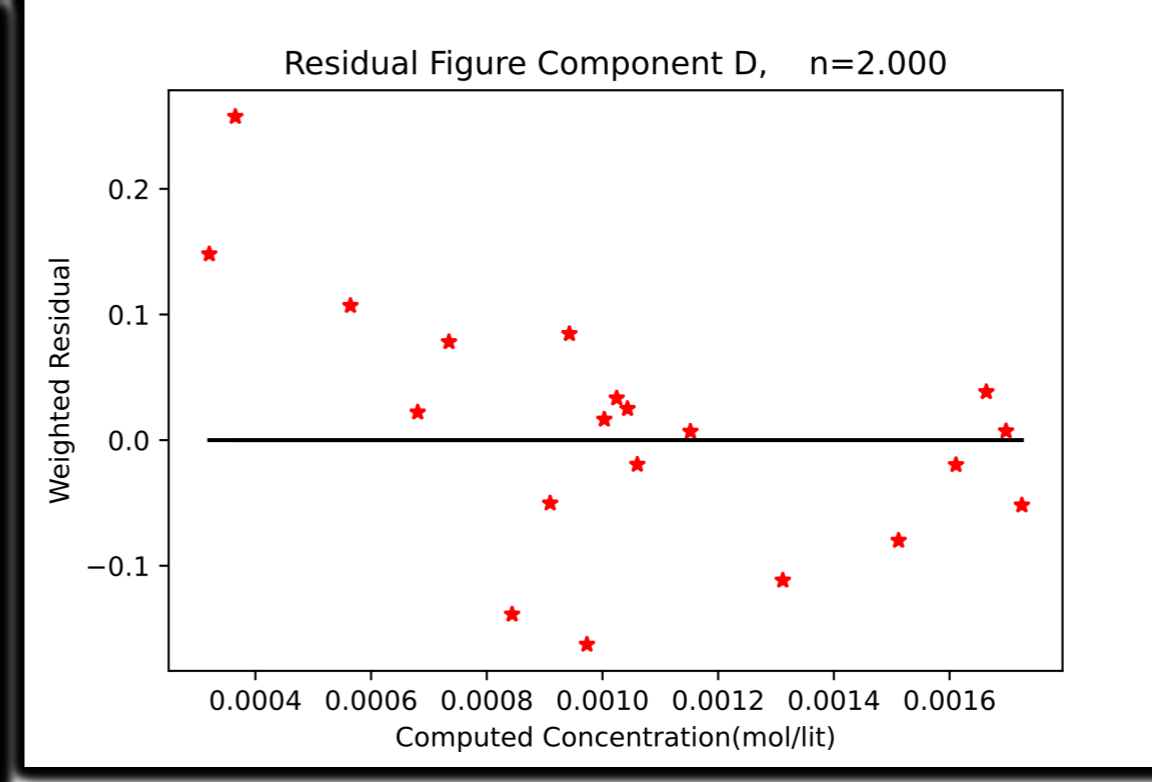
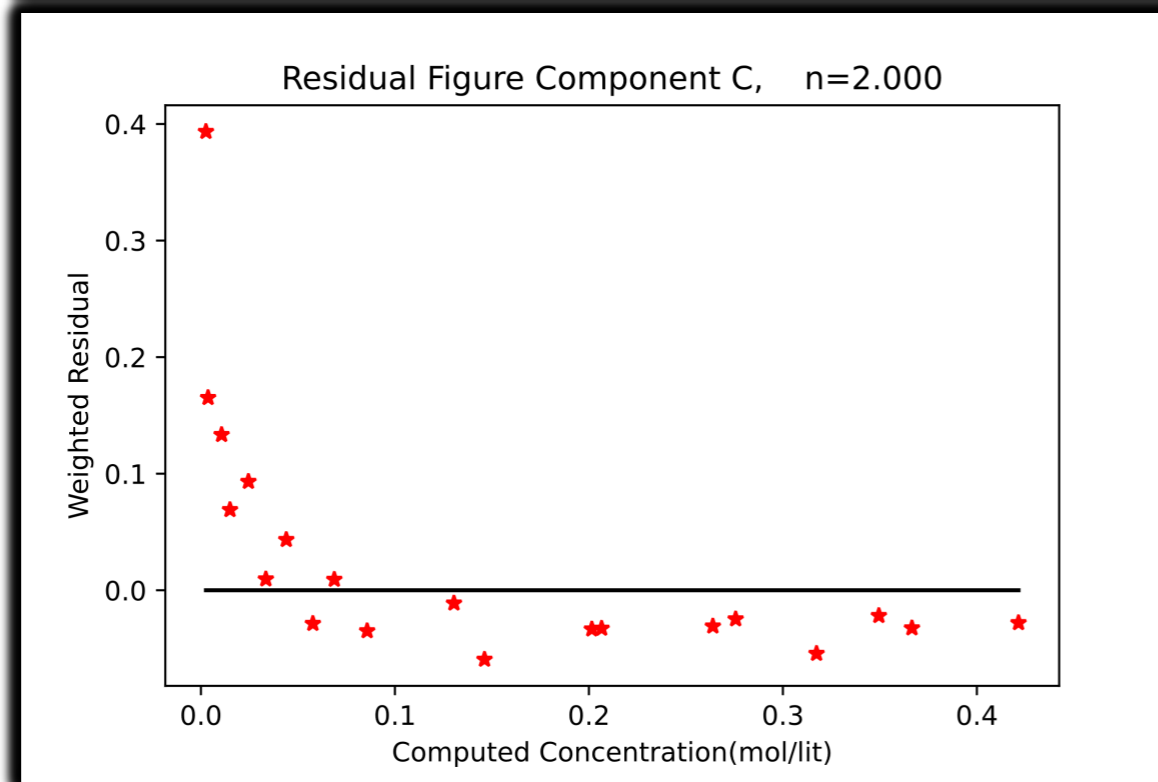
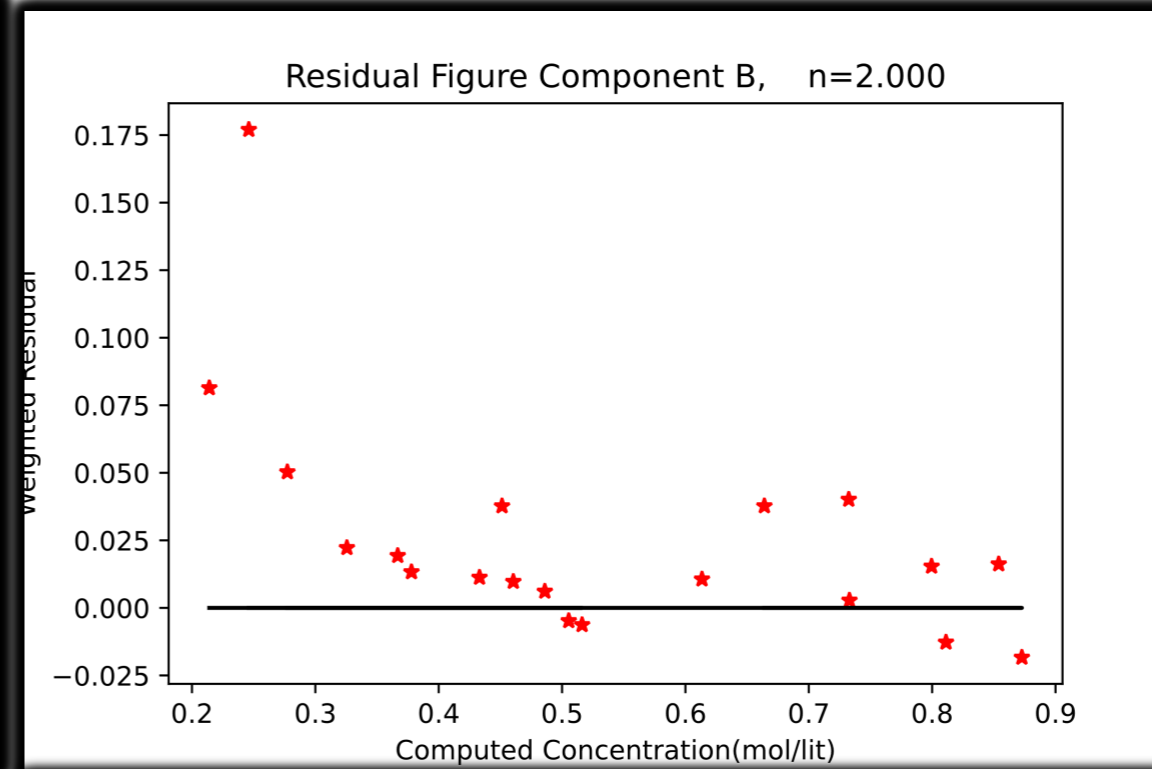
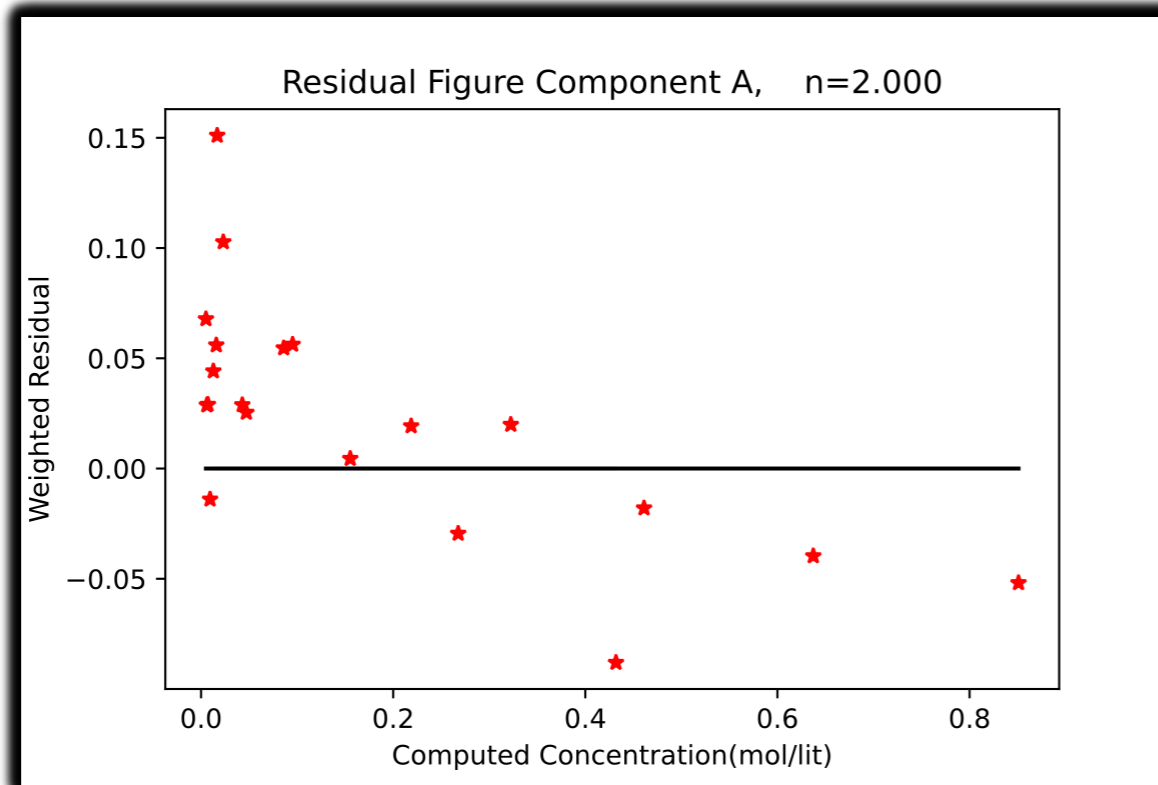
part1: batch reactor + isothermal data (result)



part1: batch reactor + isothermal data (result)

parameter/diagram	outputs		
k values	$k_1=0.01016$	$k_2=0.00195$	$k_3=1.49 \cdot 10^{-5}$
F test	$2.1 \cdot 10^4 > 2.72$		✓
T test	$91 > 2$ ✓	$50 > 2$ ✓	$50 > 2$ ✓
Independent parameters?	✓	✓	✓
parity diagram	✓		
residual diagram	✓		

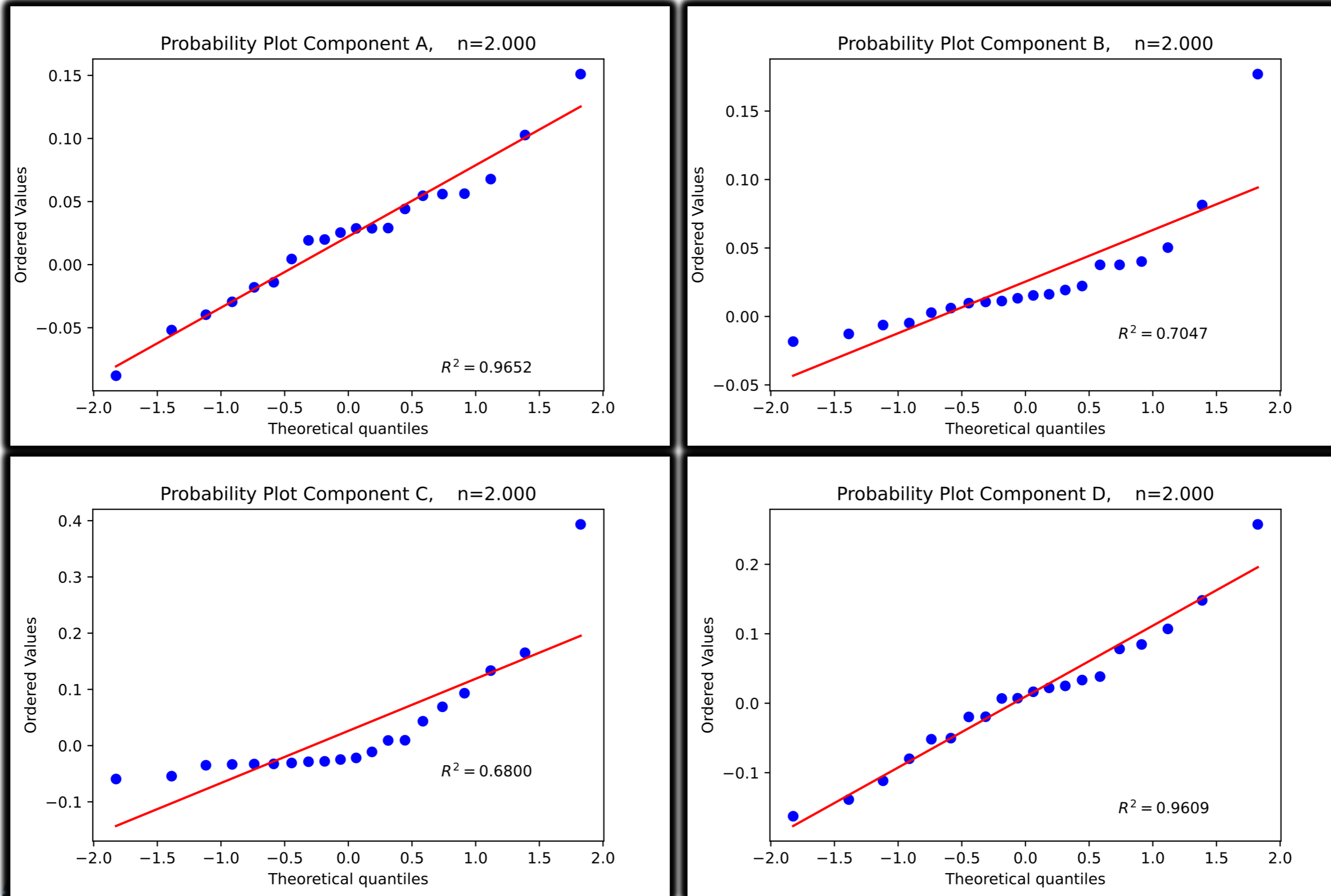
part1: batch reactor + isothermal data (result)



part1: batch reactor + isothermal data (result)

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k values	$k_1=0.01016$	$k_2=0.00195$	$k_3=1.49 \cdot 10^{-5}$
F test	$2.1 \cdot 10^4 > 2.72$		✓
T test	$91 > 2$ ✓	$50 > 2$ ✓	$50 > 2$ ✓
Independent parameters?	✓	✓	✓
parity diagram	✓		
residual diagram	✓		
Normal probability diagram	✓		

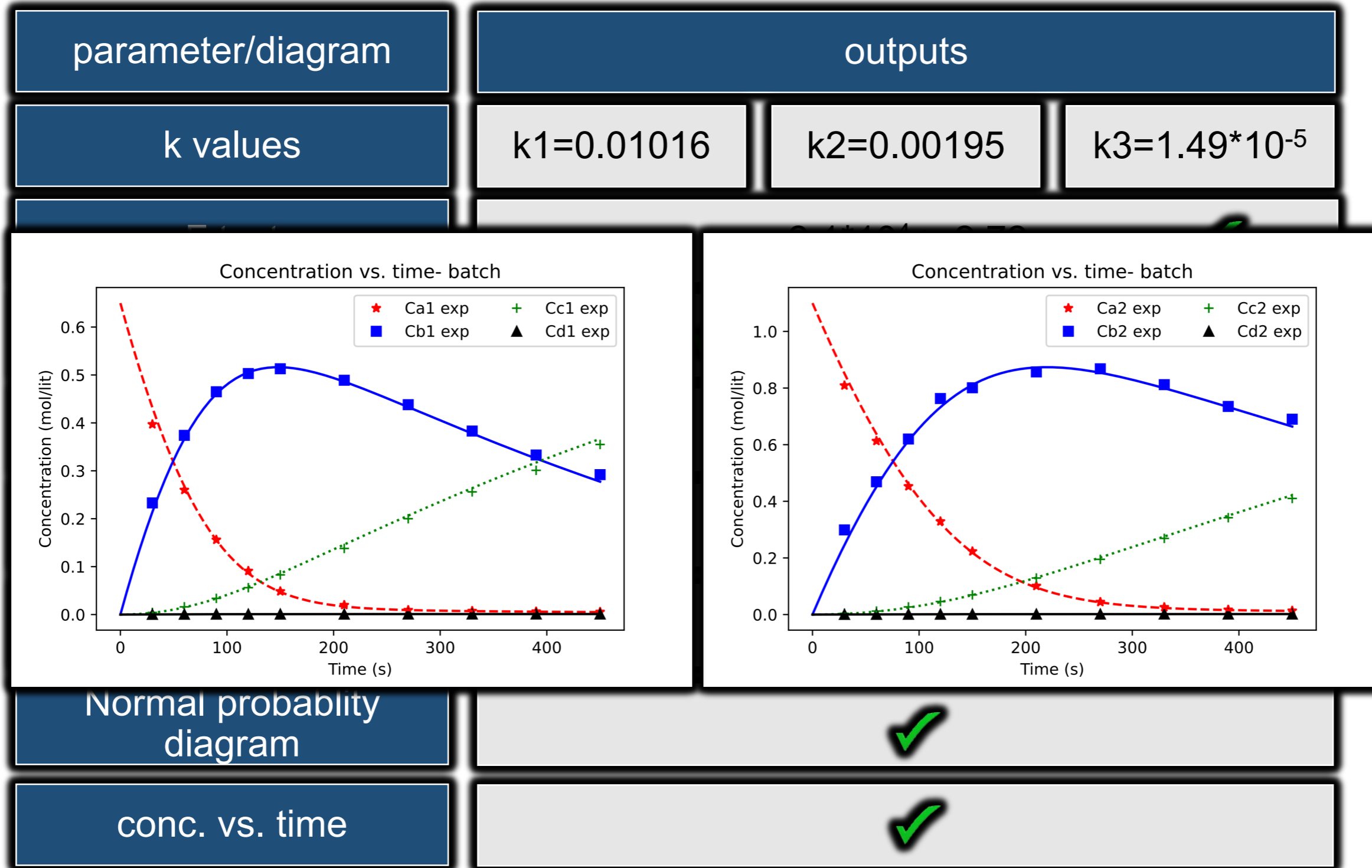
part1: batch reactor + isothermal data (result)



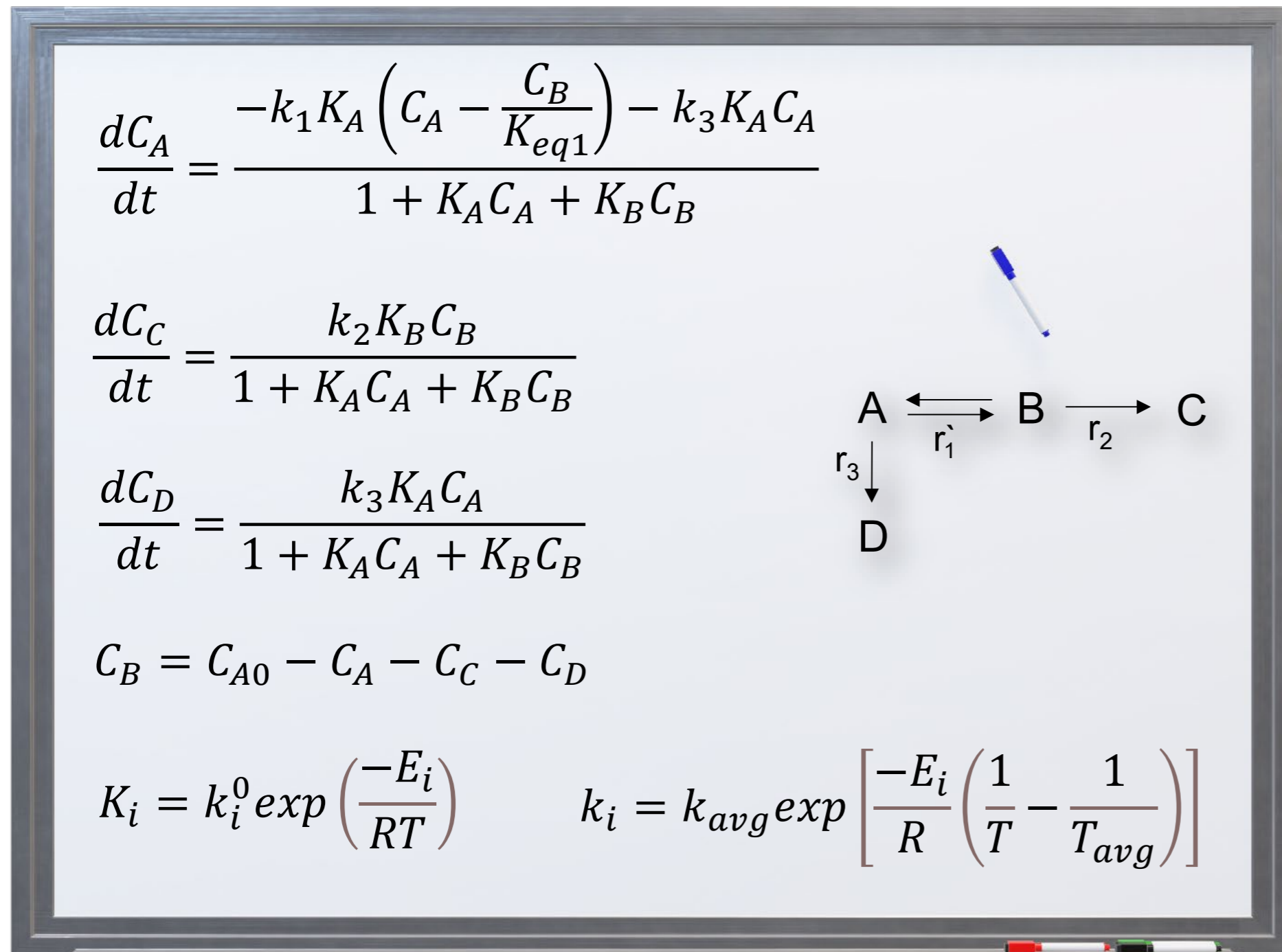
part1: batch reactor + isothermal data (result)

parameter/diagram	outputs		
k values	$k_1=0.01016$	$k_2=0.00195$	$k_3=1.49 \cdot 10^{-5}$
F test	$2.1 \cdot 10^4 > 2.72$		✓
T test	$91 > 2$ ✓	$50 > 2$ ✓	$50 > 2$ ✓
Independent parameters?	✓	✓	✓
parity diagram	✓		
residual diagram	✓		
Normal probability diagram	✓		
conc. vs. time	✓		

part1: batch reactor + isothermal data (result)



part2: batch reactor + nonisothermal data

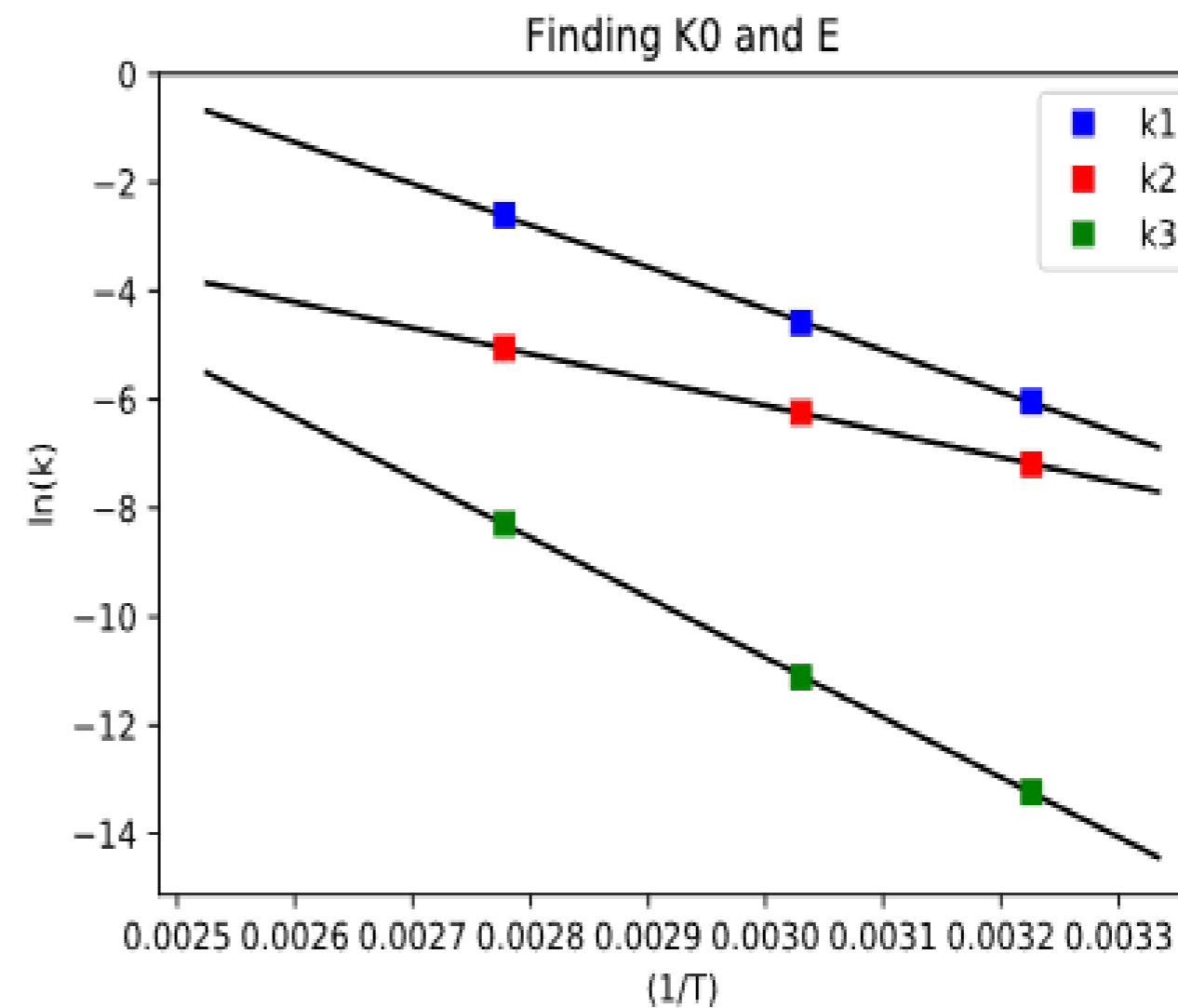
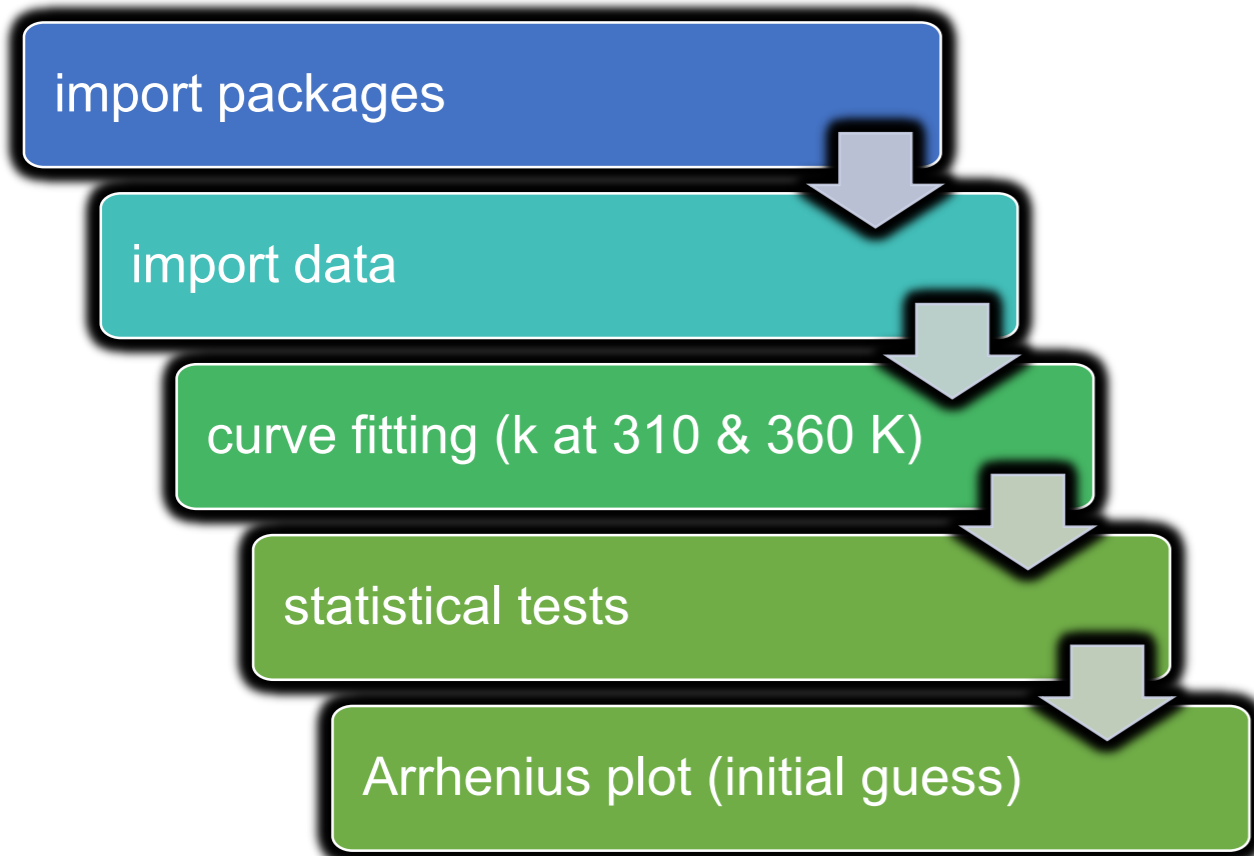


conc. at T=310, 330,
and 360 K

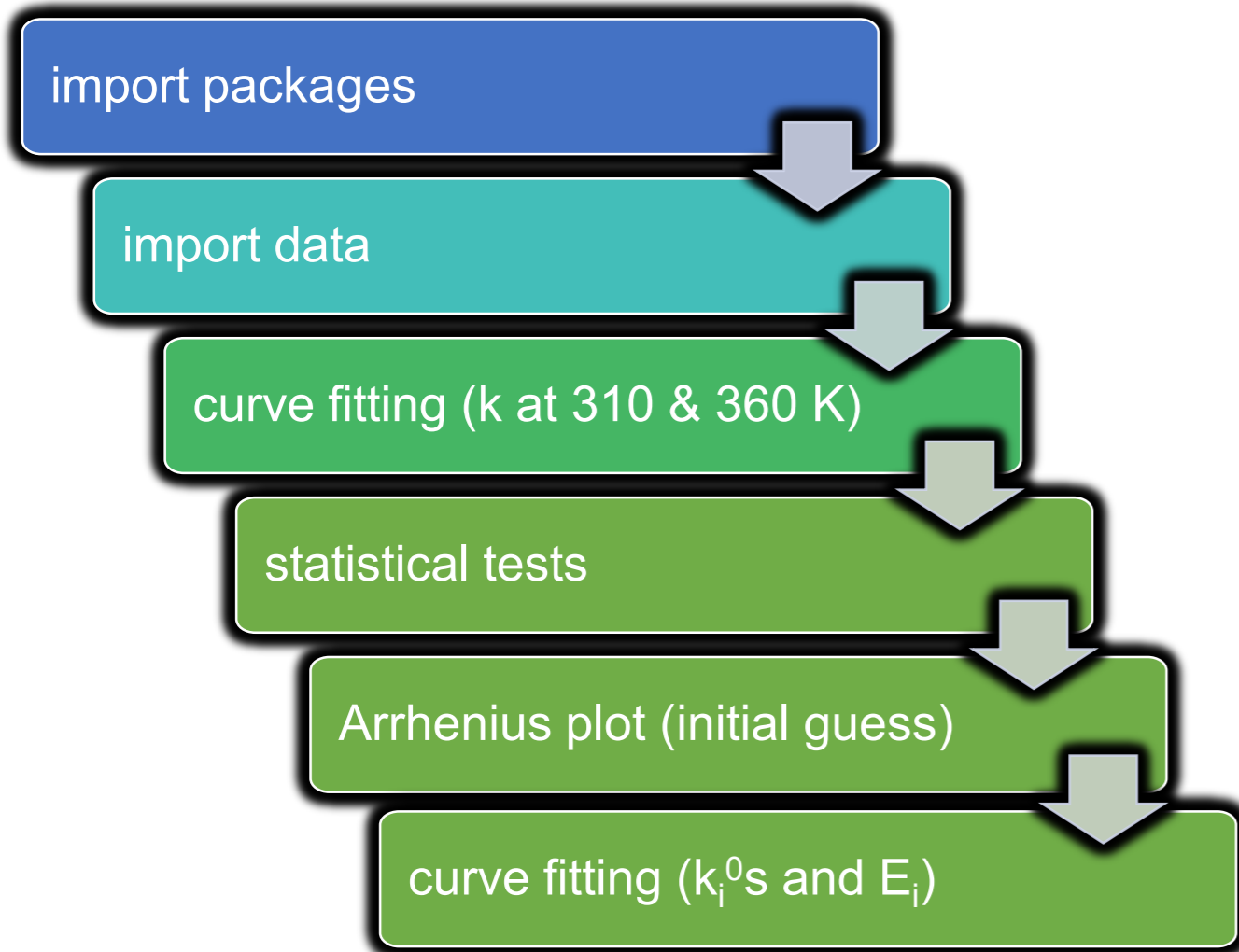
weighted regression

$k_1^0, k_2^0, k_3^0, E_1, E_2,$
 E_3

part2: batch reactor + nonisothermal data (code)



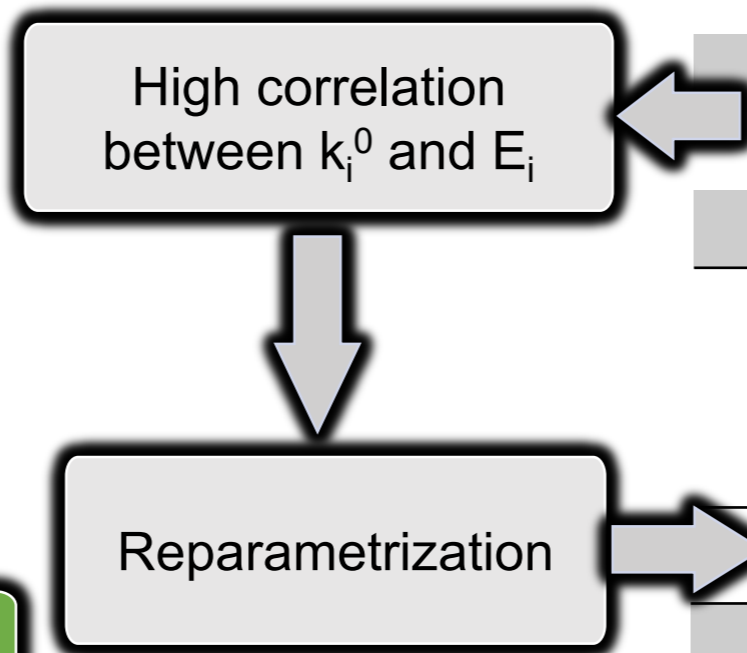
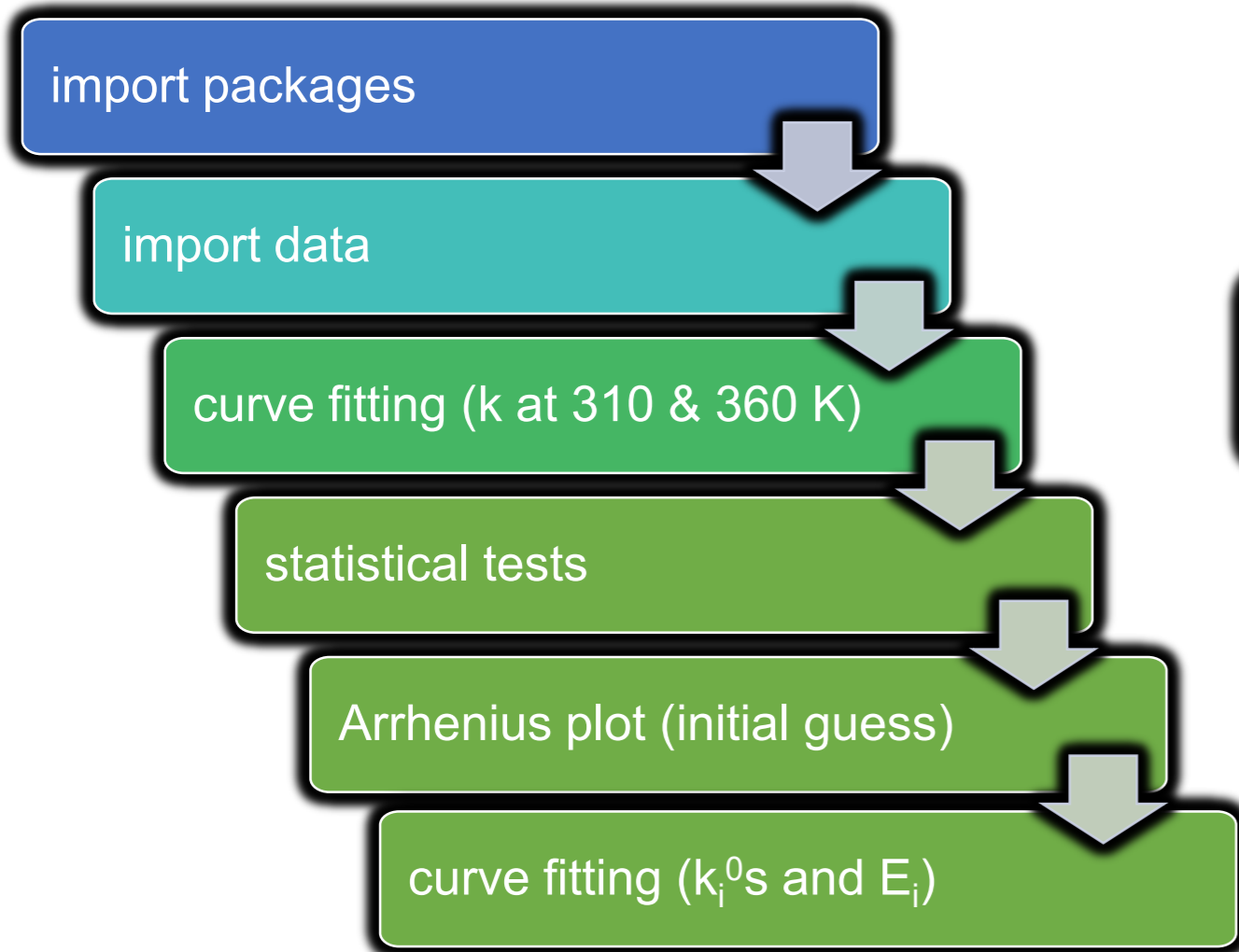
part2: batch reactor + nonisothermal data (code)



correlation coefficient matrix

	k_1^0	K_2^0	K_3^0	E_1	E_2	E_3
k_1^0	1.000	-0.416	0.396	0.999	-0.414	0.386
K_2^0	-0.416	1.000	-0.114	-0.429	0.999	-0.115
K_3^0	0.396	-0.114	1.000	0.392	-0.112	0.999
E_1	0.999	-0.429	0.392	1.000	-0.427	0.383
E_2	-0.414	0.999	-0.112	-0.427	1.000	-0.114
E_3	0.386	-0.115	0.999	0.383	-0.114	1.000

part2: batch reactor + nonisothermal data (code)



correlation coefficient matrix

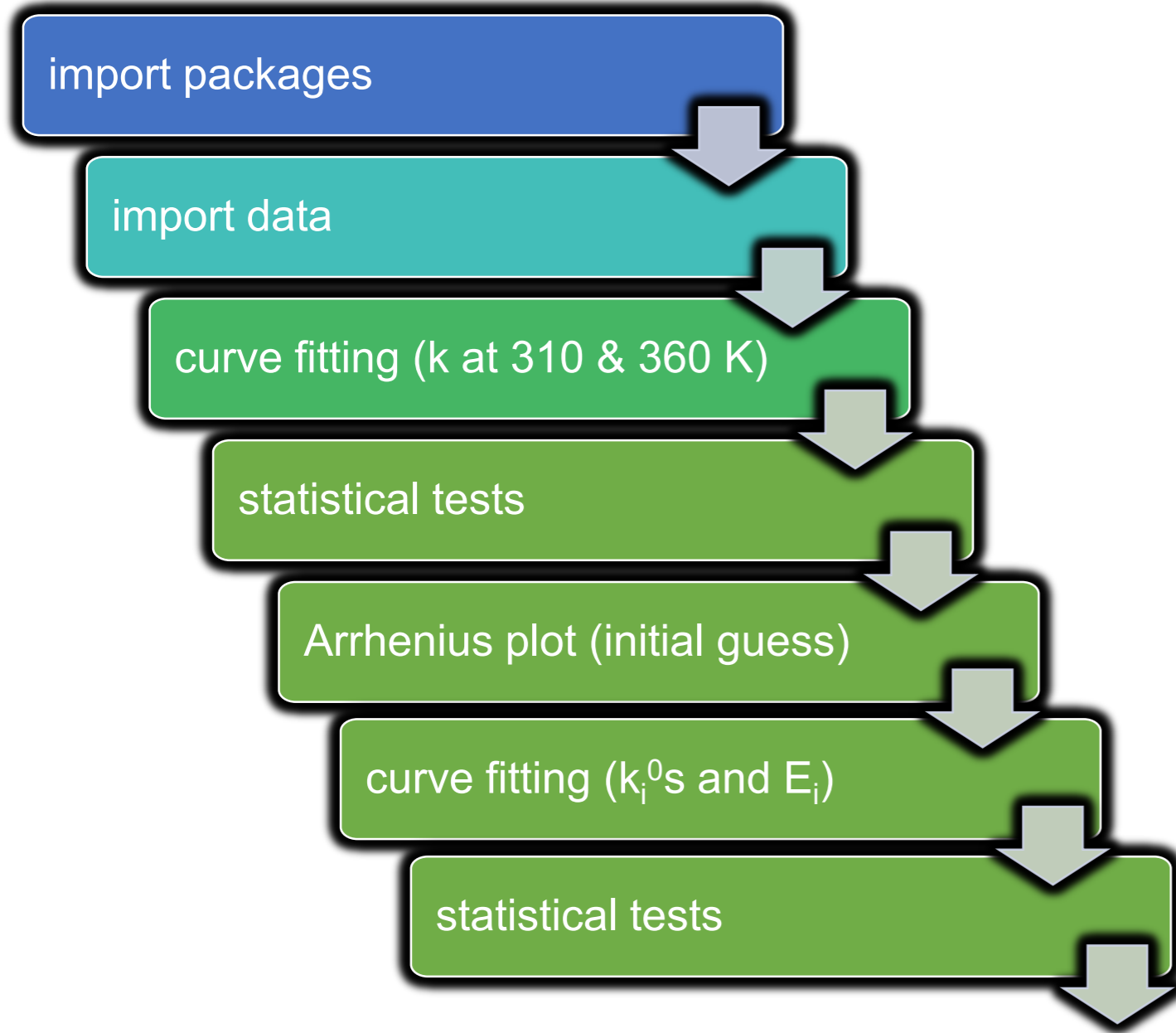
	k_1^0	K_2^0	K_3^0	E_1	E_2	E_3
k_1^0	1.000	-0.416	0.396	0.999	-0.414	0.386
K_2^0	-0.416	1.000	-0.114	-0.429	0.999	-0.115
K_3^0	0.396	-0.114	1.000	0.392	-0.112	0.999
E_1	0.999	-0.429	0.392	1.000	-0.427	0.383
E_2	-0.414	0.999	-0.112	-0.427	1.000	-0.114
E_3	0.386	-0.115	0.999	0.383	-0.114	1.000

$$k_i = k_{avg} \exp \left[\frac{-E_i}{R} \left(\frac{1}{T} - \frac{1}{T_{avg}} \right) \right]$$

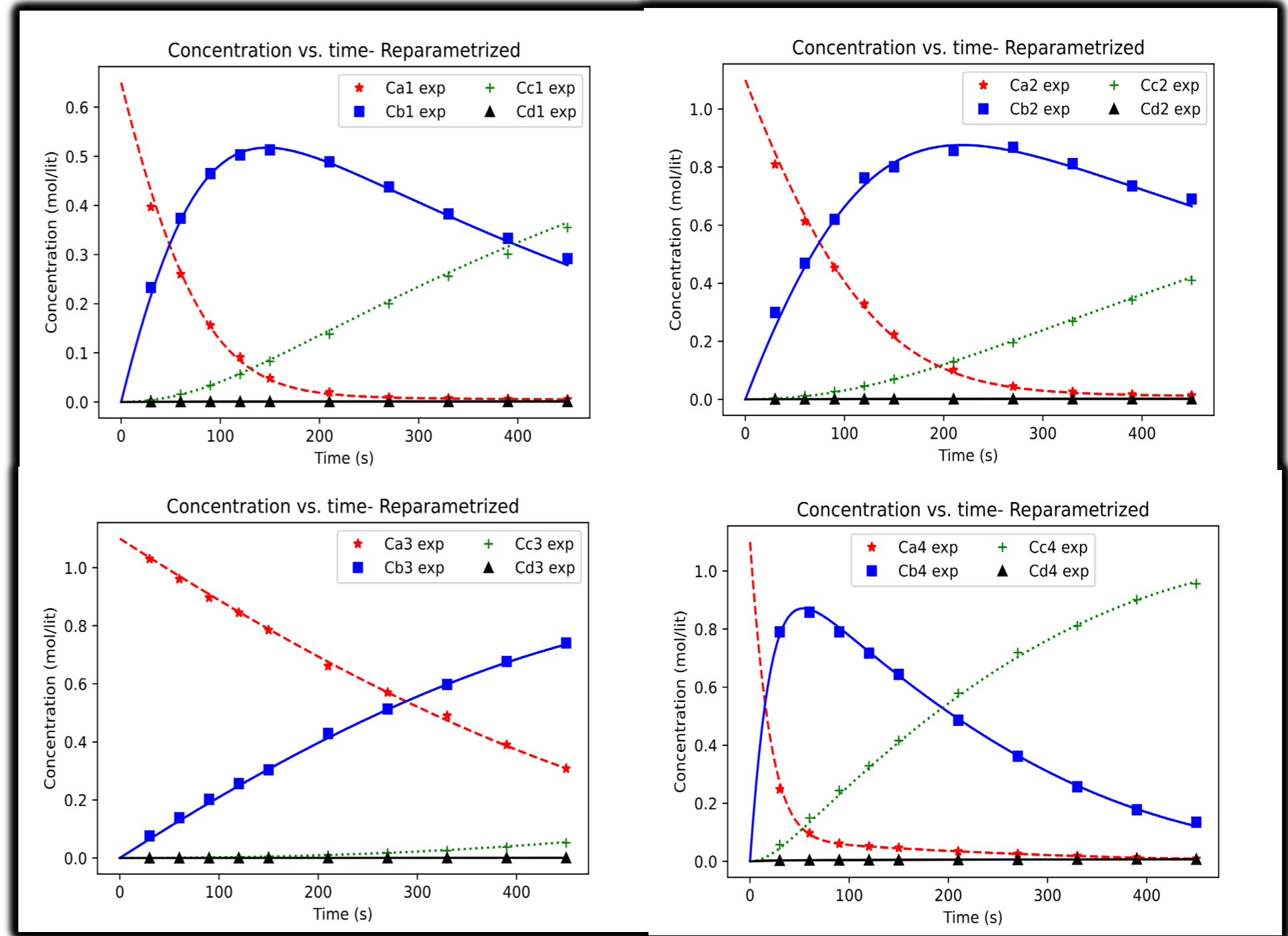
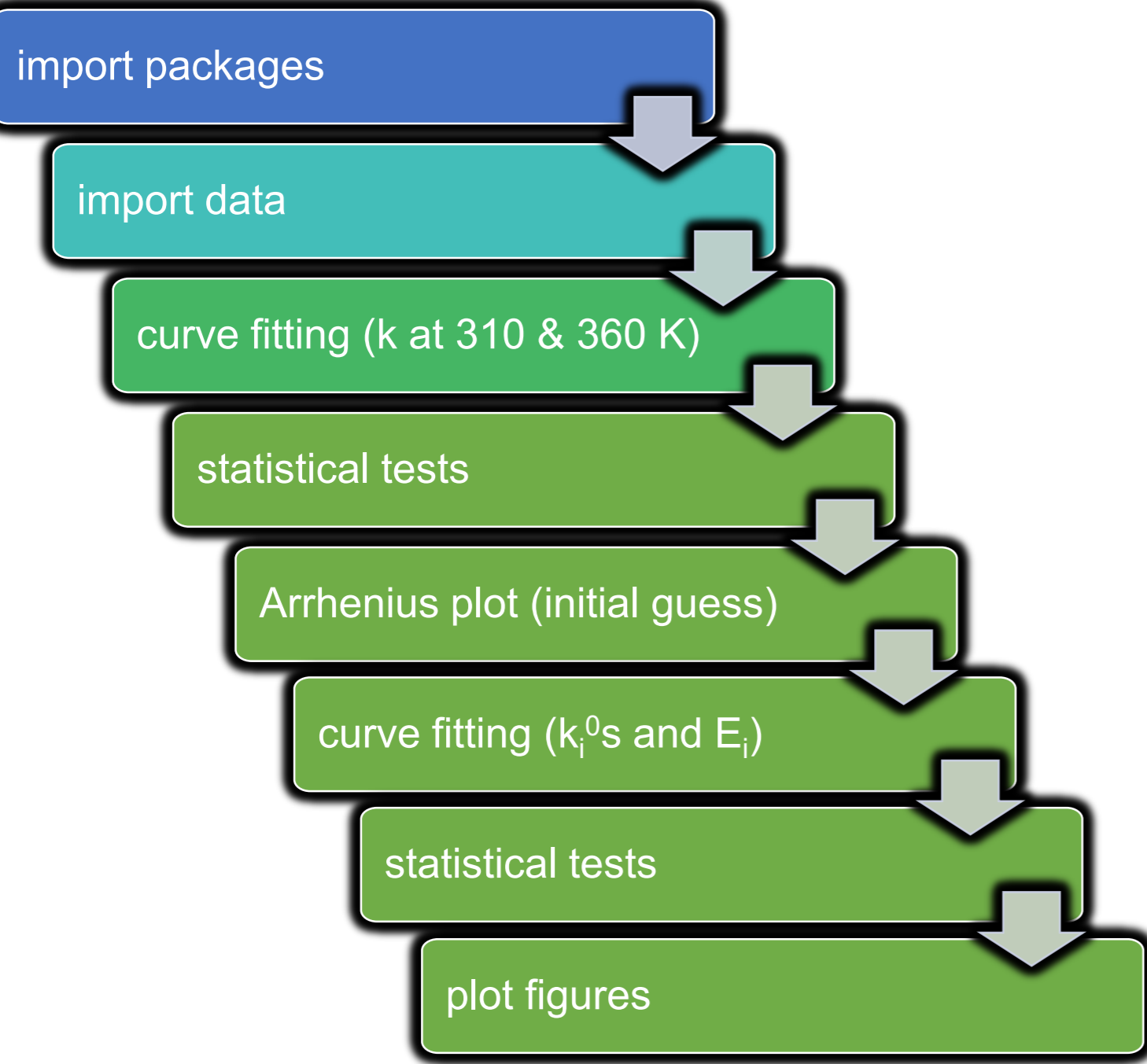
correlation coefficient matrix

	k_1^0	K_2^0	K_3^0	E_1	E_2	E_3
k_1^0	1.000	-0.356	0.408	0.401	0.135	0.239
K_2^0	-0.356	1.000	-0.099	0.225	-0.609	0.045
K_3^0	0.408	-0.099	1.000	0.263	0.007	0.200
E_1	0.401	0.225	0.263	1.000	-0.427	0.377
E_2	0.135	-0.609	0.007	-0.427	1.000	-0.120
E_3	0.239	0.045	0.200	0.377	-0.120	1.000

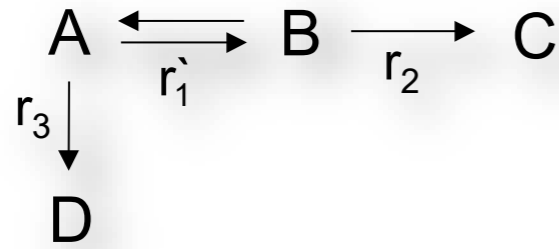
part2: batch reactor + nonisothermal data (code)



part2: batch reactor + nonisothermal data (code)



part3: CSTR isothermal data+ all data

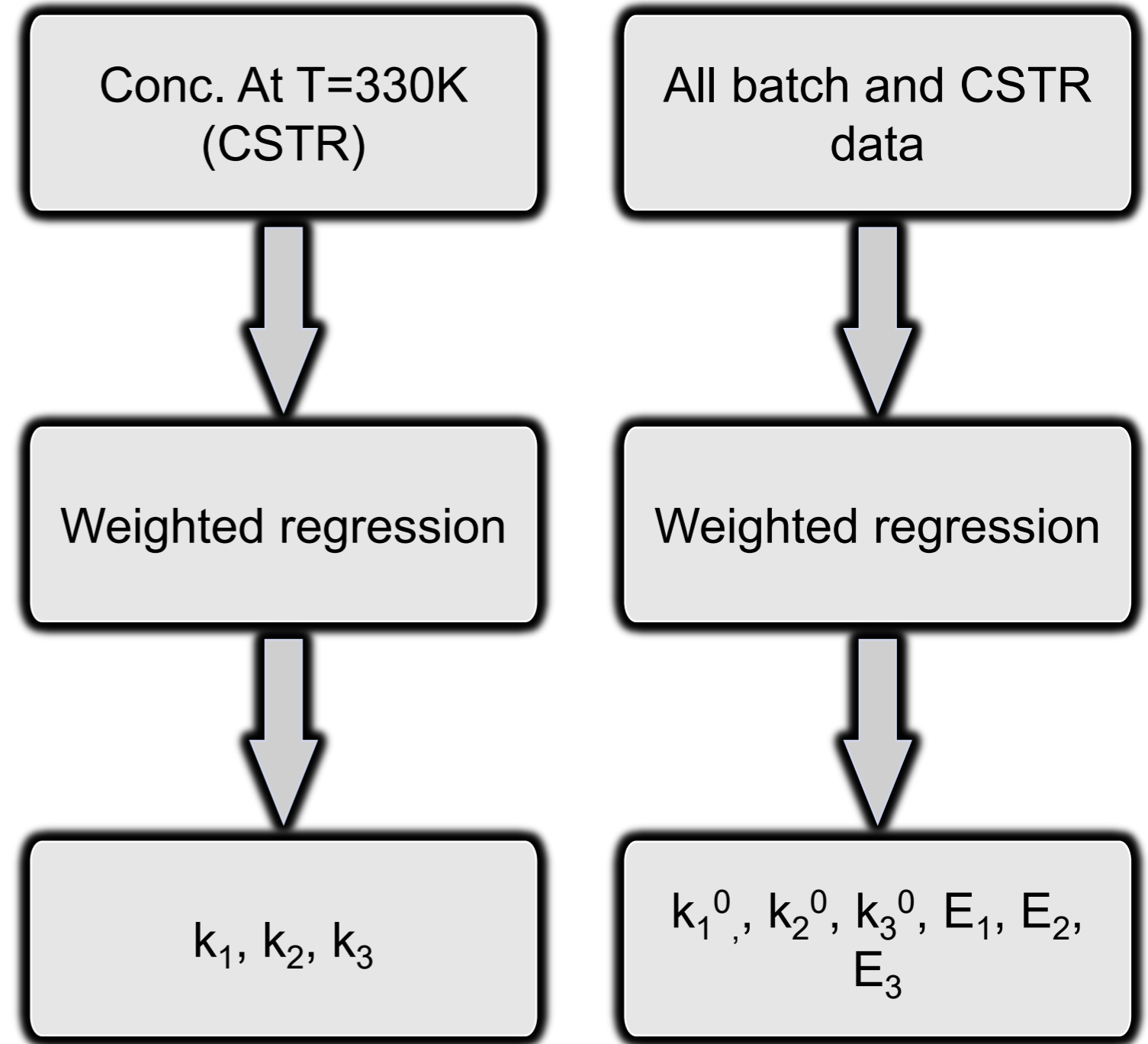


$$C_A = C_{A0} \frac{\left[-k_1 K_A \left(C_A - \frac{C_B}{K_{eq1}} \right) - k_3 K_A C_A \right] \tau}{1 + K_A C_A + K_B C_B}$$

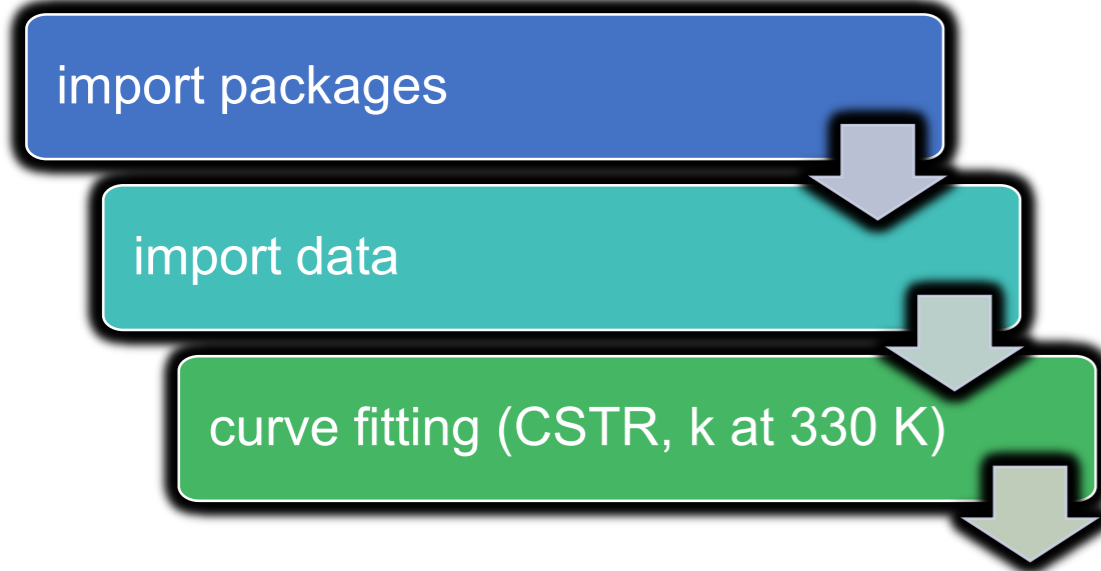
$$C_C = \frac{(k_2 K_B C_B) \tau}{1 + K_A C_A + K_B C_B}$$

$$C_D = \frac{(k_3 K_A C_A) \tau}{1 + K_A C_A + K_B C_B}$$

$$C_B = C_{A0} - C_A - C_C - C_D$$



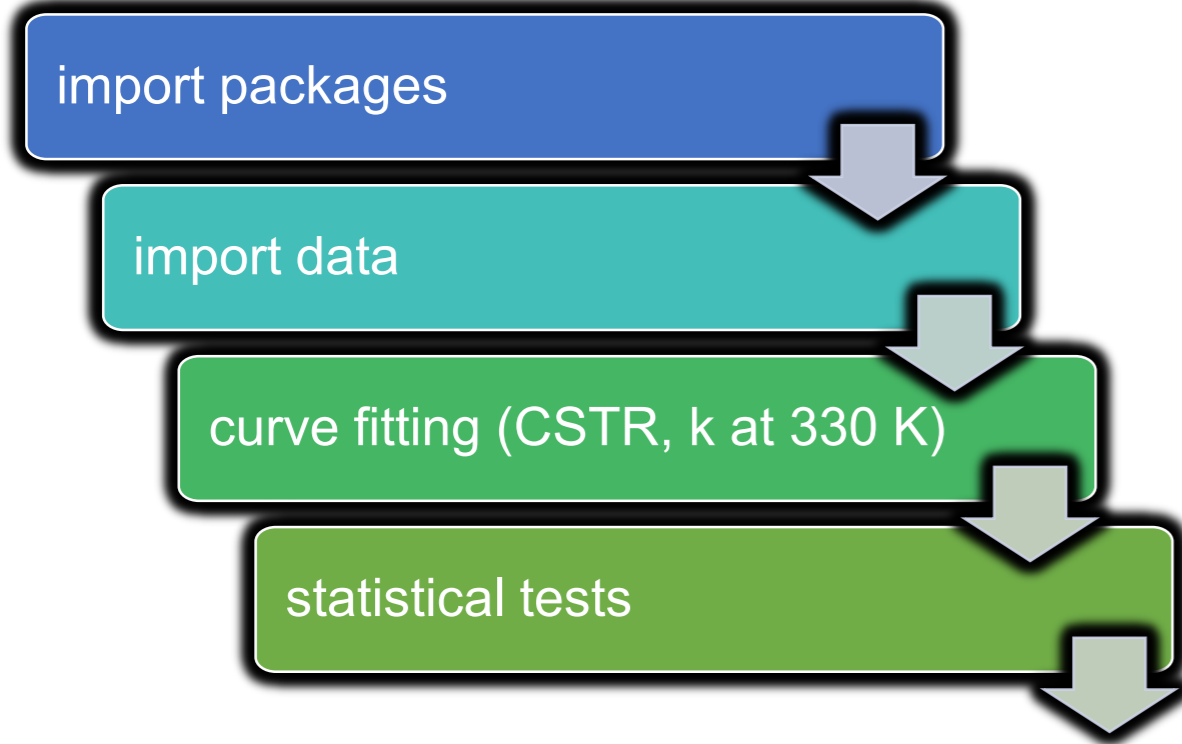
part3: CSTR isothermal data+ all data (code)



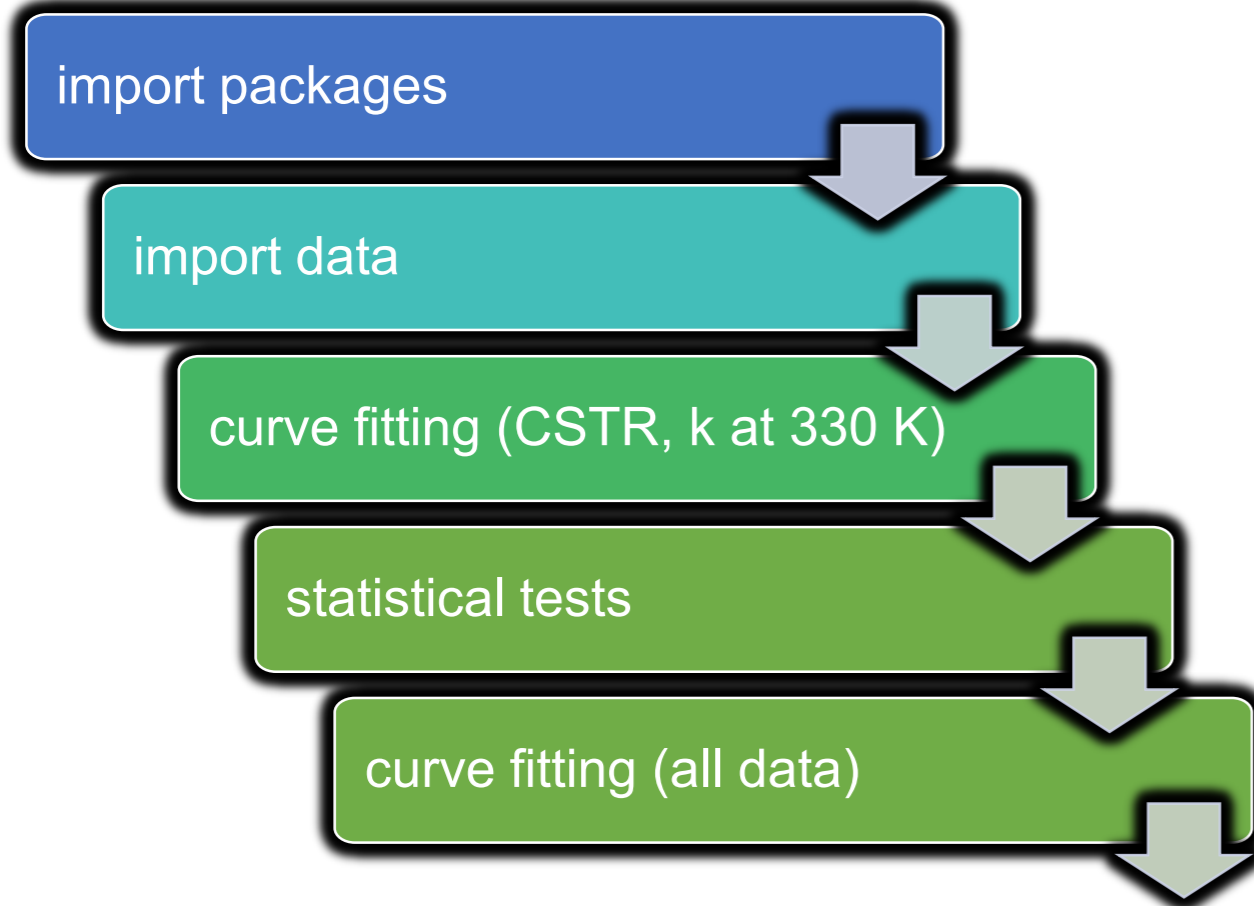
Systems of algebraic equations

```
91 def model(res_t,k1,k2,k3):
92
93     C_sol=np.zeros((len(res_t),4))
94     C0=np.zeros((len(res_t),4))
95     C0=np.c_[C_A0,C0[:,1],C0[:,2],C0[:,3]]
96
97     # fsolve find solutions for nonlinear systems of equations
98     for i in range(len(C0)):
99         Guess=C0[i,:]
100         C_sol[i,:]=fsolve(rxn,Guess,args=(res_t[i],C_A0[i],k1,k2,k3))
101
102     return C_sol.ravel(order='F')
```

part3: CSTR isothermal data+ all data (code)



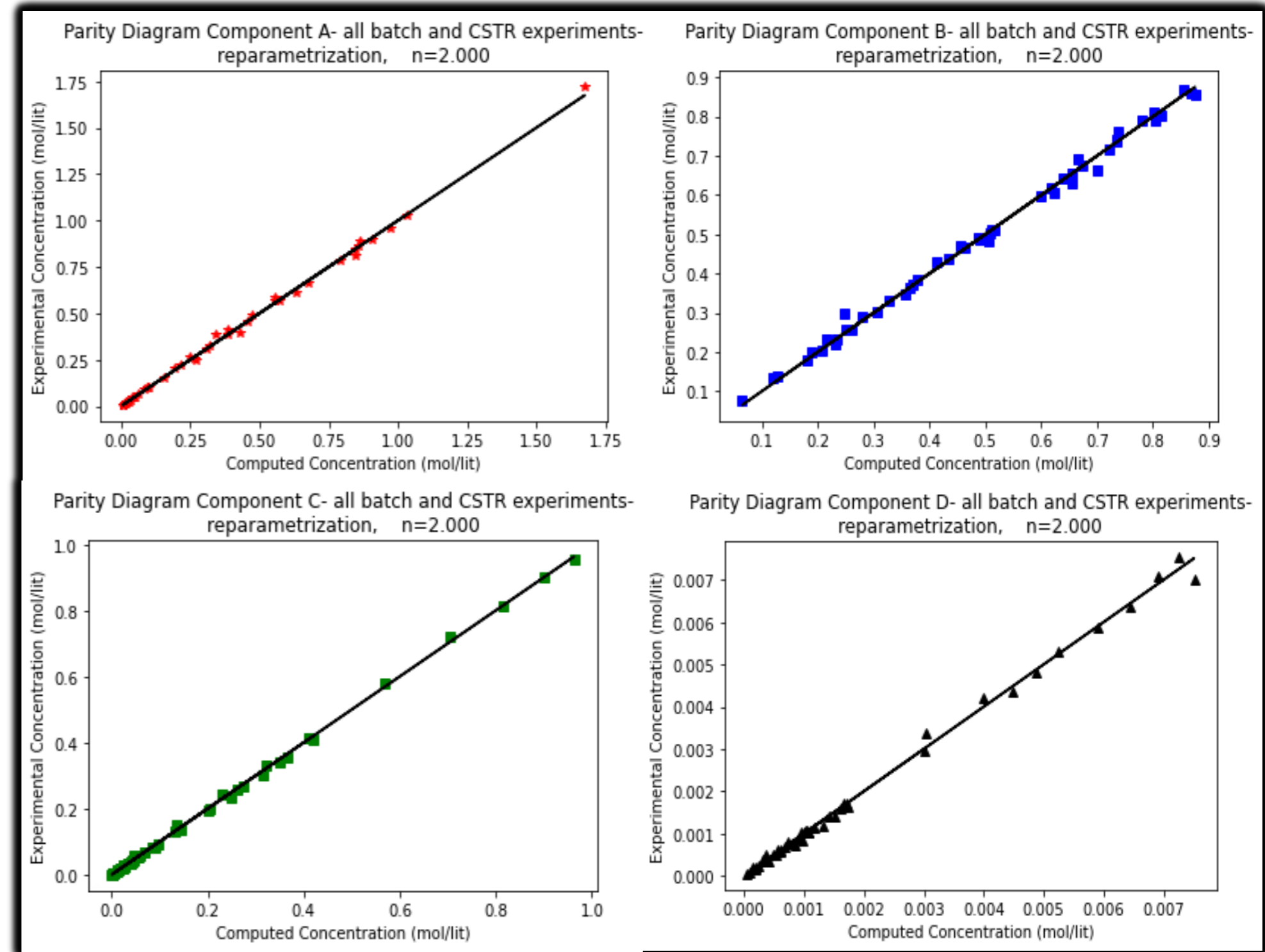
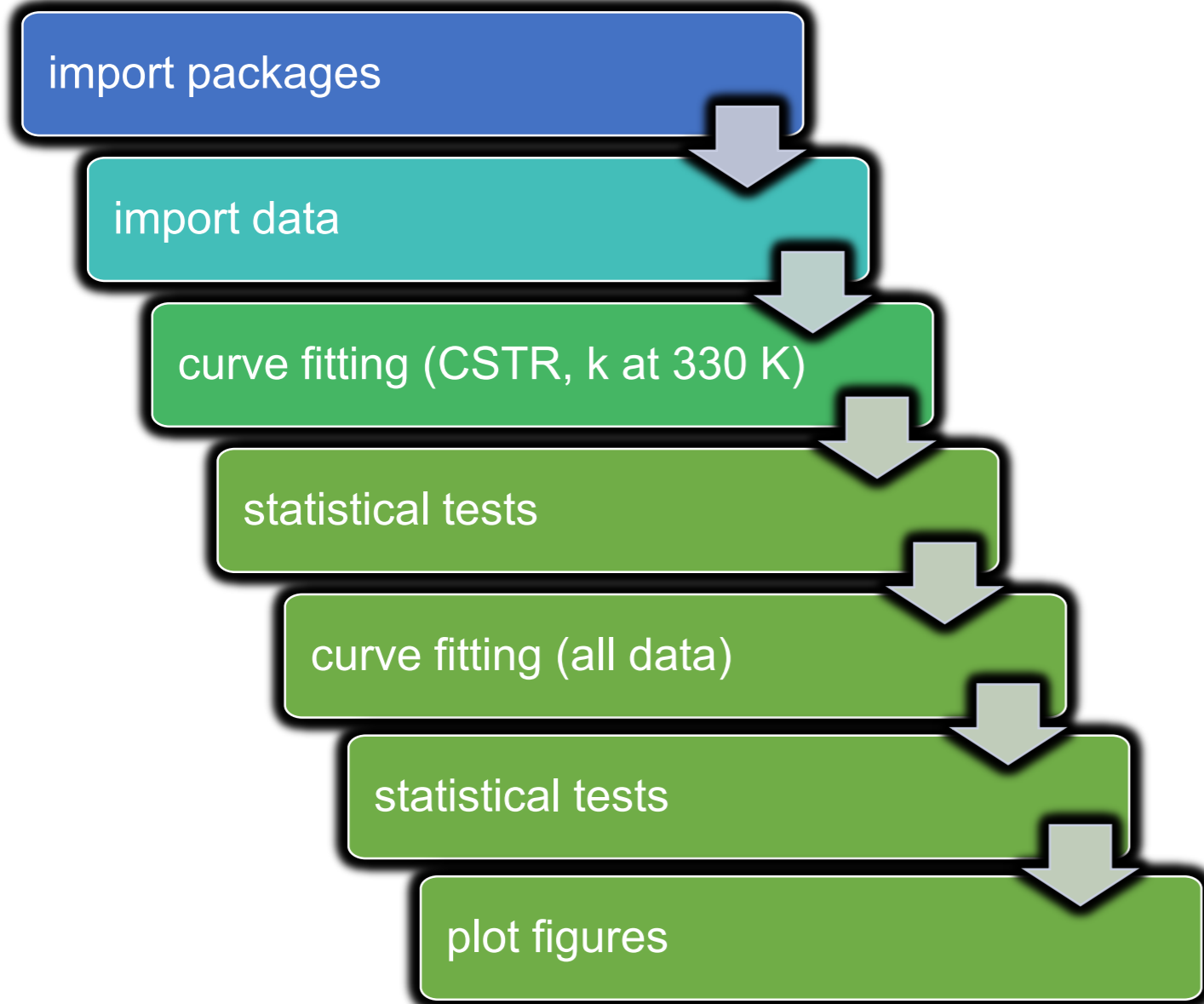
part3: CSTR isothermal data+ all data (code)



ODE + Algebraic equations

```
144 def model_ABCS_mod(t_All,k_avg1,k_avg2,k_avg3,E1,E2,E3):
145
146     #add t=0 to the Time vector-because the ODEint needs the initial values.
147     tt_All=np.array([0])
148     tt_All=np.append(tt_All,t_All)
149
150     C_sol_C=np.zeros((len(res_t_All),4))
151     C0_C=np.zeros((len(res_t_All),4))
152     C0_C=np.c_[C_A0_All,C0_C[:,1],C0_C[:,2],C0_C[:,3]]
153
154     #Find CSTR concentrations
155     for i in range(len(C0_C)):
156         Guess_C=C0_C[i,:]
157         C_sol_C[i,:]=fsolve(rxn_C_mod,Guess_C,args=(tt_All[i+1],C_A0_All[i],\
158             k_avg1,k_avg2,k_avg3,E1,E2,E3,T_ABCS[i+4],KA_ABCS[i+4],\
159             KB_ABCS[i+4],Keq1_ABCS[i+4]))
160     #calculate Batch concentrations
161     C0_B=np.array([[0.65,0,0,0],[1.1,0,0,0],[1.1,0,0,0],[1.1,0,0,0]])
162     C_sol_B=np.zeros((len(t)+1,np.size(C0_B)))
163     Cb_B=np.zeros((len(t)+1,len(T_ABCS[0:4])))
164
165     for i in range(len(T_ABCS[0:4])):
166         C_sol_B[:,i*4:i*4+4]=odeint(rxn_B_mod,C0_B[i,:],tt_All[0:11],\
167             args=(k_avg1,k_avg2,k_avg3,E1,E2,E3,T_ABCS[i],\
168             KA_ABCS[i],KB_ABCS[i],Keq1_ABCS[i]))
169         Cb_B[:,i]=C0_B[i,0]-C_sol_B[:,i*4]-C_sol_B[:,i*4+2]-C_sol_B[:,i*4+3]
170         C_sol_B[:,i*4:i*4+4]=np.c_[C_sol_B[:,i*4],Cb_B[:,i],C_sol_B[:,i*4+2]\
171             ,C_sol_B[:,i*4+3]]
172     C_sol_B=C_sol_B[1:,:]
173     C_sol_B=C_sol_B.ravel(order='F')
174     C_sol_ABCS=np.append(C_sol_B,C_sol_C.ravel(order='F'))
175
176     #return to curve fit function
177     return C_sol_ABCS
178
```

part3: CSTR isothermal data+ all data (code)



overview

- ✓ introduction to Python
- ✓ How to work with Python?
- ✓ Python for numerical programming
- ✓ EUROKIN parameter estimation-case 1
- complex system
- summary

complex system

mass transport equations

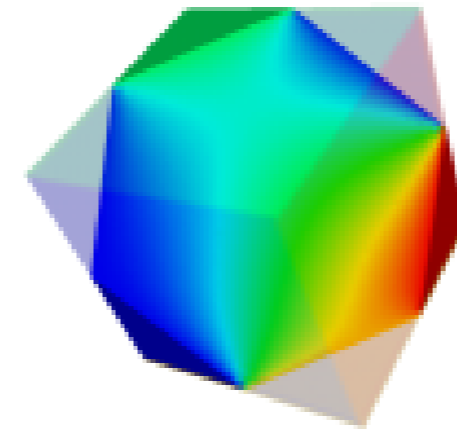
heat transport equations

momentum transfer equations

reaction



FENICS
PROJECT



SfePy

overview

- ✓ introduction to Python
- ✓ How to work with Python?
- ✓ Python for numerical programming
- ✓ EUROKIN parameter estimation-case 1
- ✓ complex system
- summary

summary

advantages

free

open
source

beginner-
friendly

easy to import
export, & visualize

operating system

Windows

Linux

MacOS

help

powerful

inside the software and
online

solution to equations

AE

ODE

PDE

optimization

available in Scipy package

Scipy.optimize

statistical calculation

available in Scipy package

Scipy.stats

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