



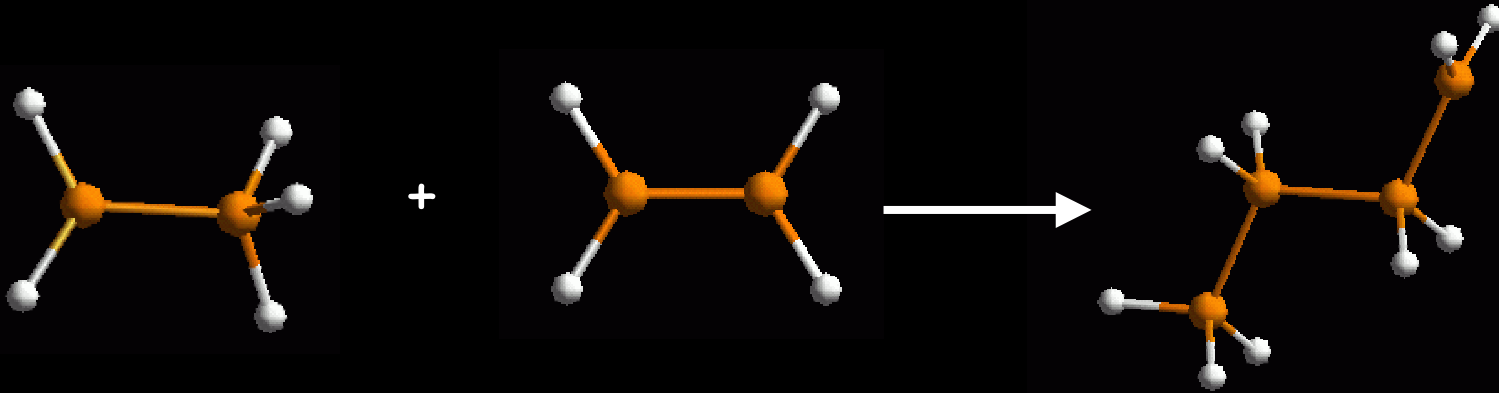
Kinetic parameters of chemical reactions  
calculated from first principles

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## Outline

- Theoretical procedures on addition of ethylradical to ethene
- Cyclization of primary ethylbenzene radical : coupled internal rotations
- Hydrogen abstraction reactions : Free rotor approach
- Application on other reaction of coke formation

# Chemical Reaction : $A+B \rightarrow C$



Experimental information :  
reaction enthalpies, reaction entropies, kinetic parameters,...

Macroscopic quantities

?

Microscopic information on chemical reaction

# Theoretical calculations

Static

Dynamic

Ab initio

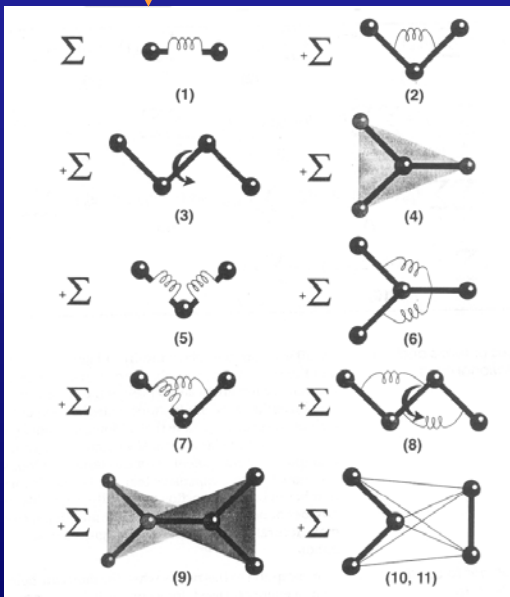
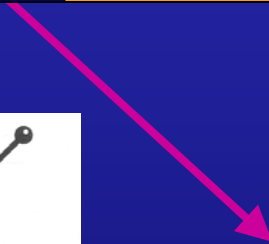
Hartree-Fock  
Density Functional Theory

Car Parrinello MD

Semi-empirical

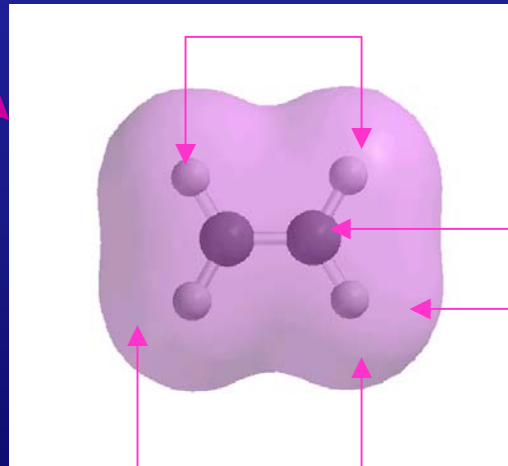
Empirical

Molecular Mechanics



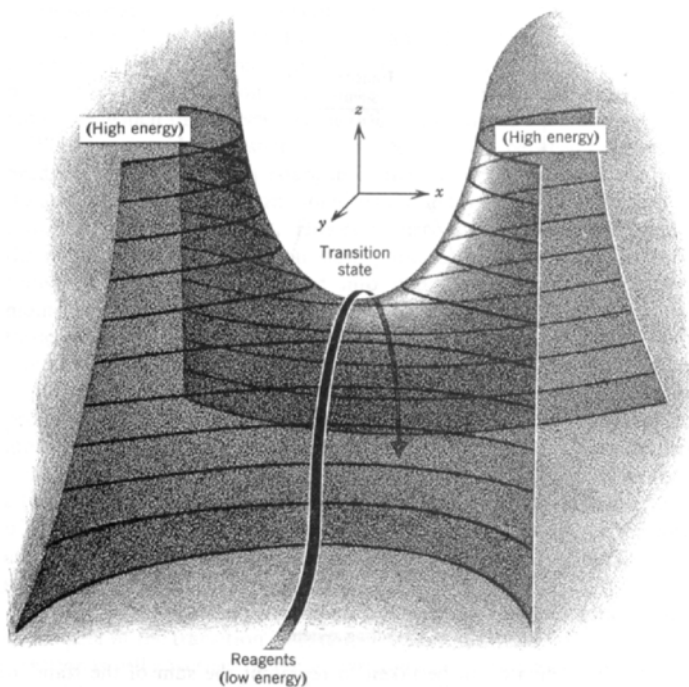
Interactions are described by empirical Force Fields

Interactions between nuclei



Interactions between electrons

Interactions between electrons

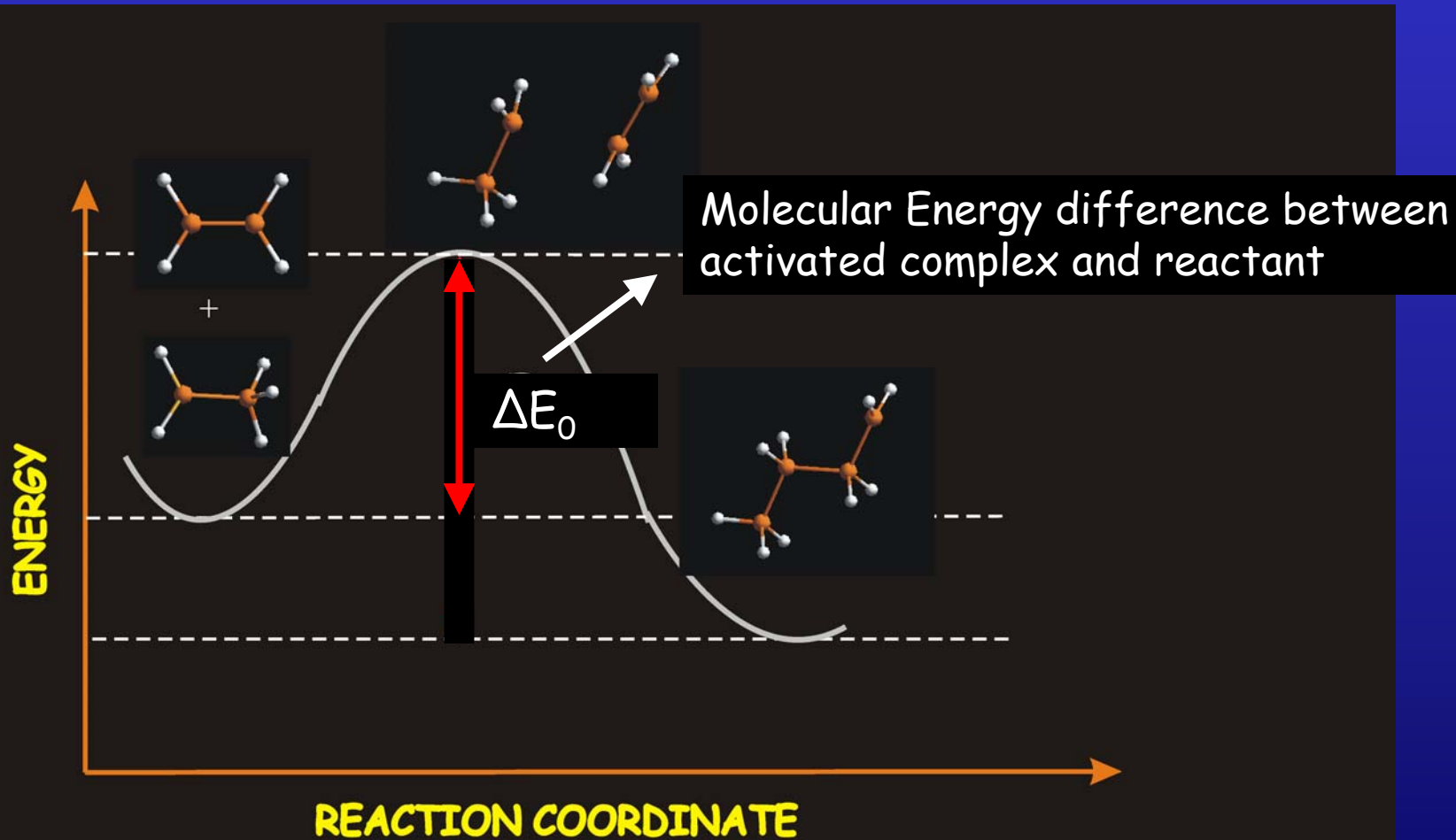


## *Transition State Theory*

Rate equation :  $k(T) = \frac{k_B T}{h} \frac{q_{\ddagger}}{q_A q_B} e^{-\frac{\Delta E_0}{k_B T}}$   $\longrightarrow$  *Molecular Energy difference at absolute zero between activated complex and reactant*

↓

*Microscopically determined partition function*



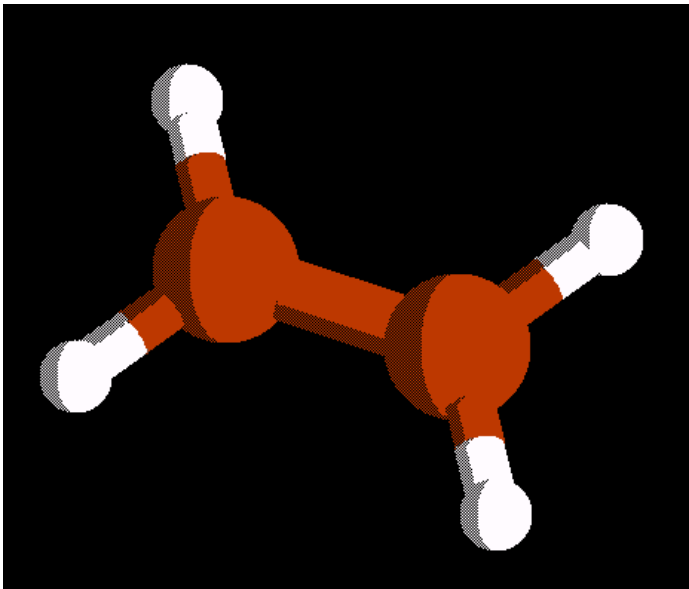
## Partition function $q$ :

$$q = q_{\text{trans}} q_{\text{vib}} q_{\text{rot}} q_{\text{electronic}}$$



- translational : global translation of the molecule
- rotational : global rotation of the molecule
- electronic = 1 (Electrons are in groundstate)
- vibrational :

$$q_{\text{vib},i} = \frac{e^{-\frac{h\nu_i}{2k_B T}}}{1 - e^{-\frac{h\nu_i}{k_B T}}}$$

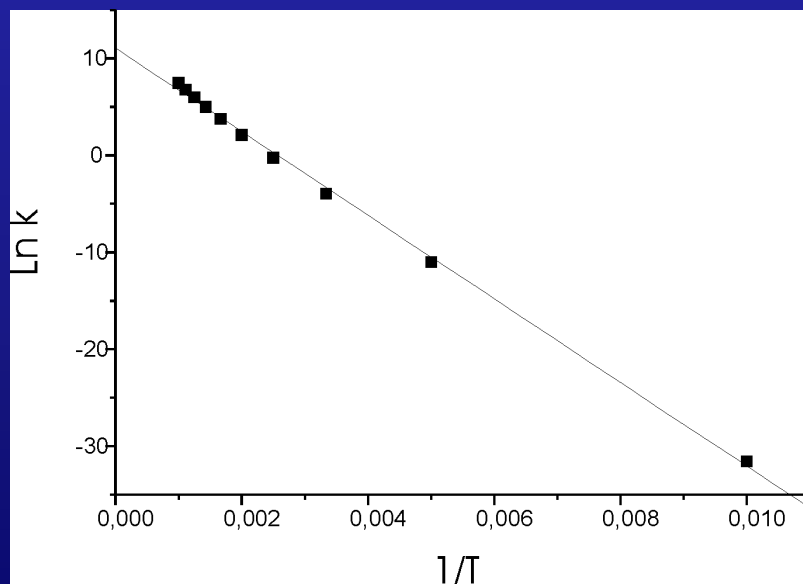


$$k(T) = \frac{k_B T}{h} \frac{q_{\ddagger}}{q_A q_B} e^{-\frac{\Delta E_0}{k_B T}}$$

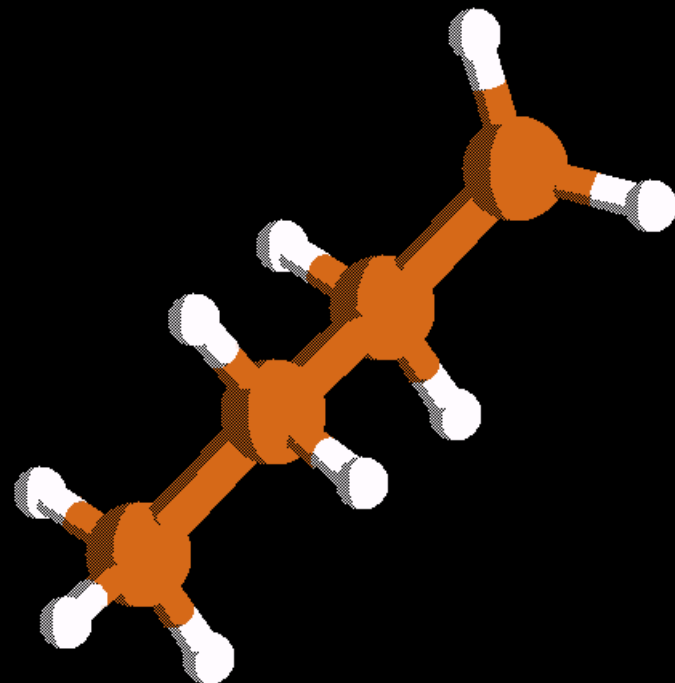


Arrhenius rate law

$$k(T) = A e^{-\frac{E_a}{RT}}$$



Exp :  $E_a = 30.5 \text{ kJ/mol}$



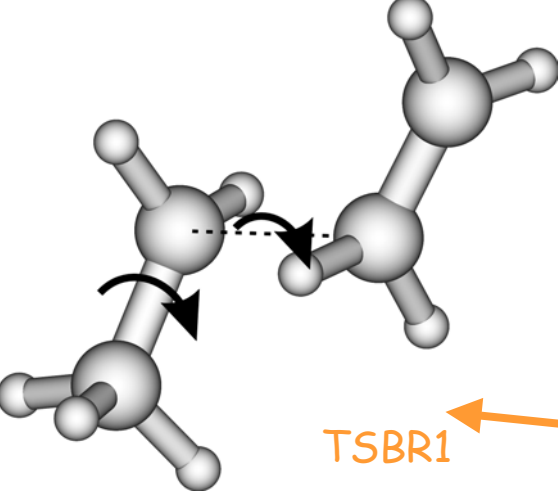
Kinetic parameters  
(DFT/B3LYP/6-311G\*\*)

$\Delta E_0 = 34.9 \text{ kJ/mol}$

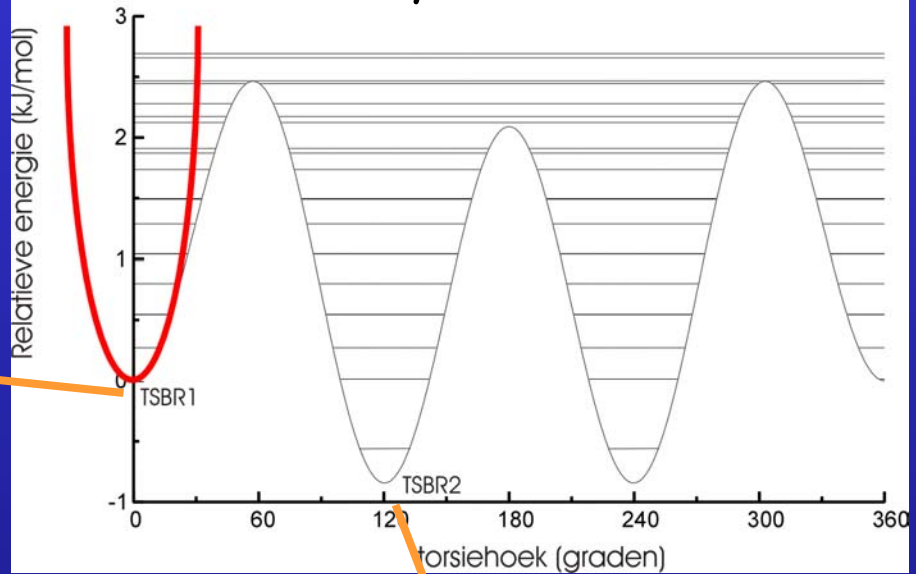
$E_a = 36.81 \text{ kJ/mol}$

$A = 9.75 \text{ E7 dm}^3/\text{mol.s}$



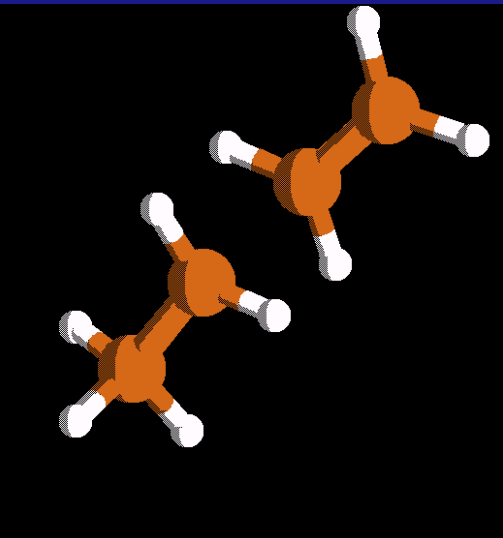


# Ethyl rotation

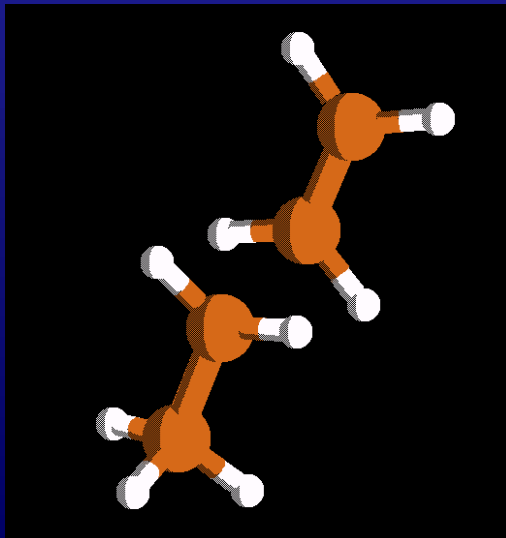


More transition states come into play

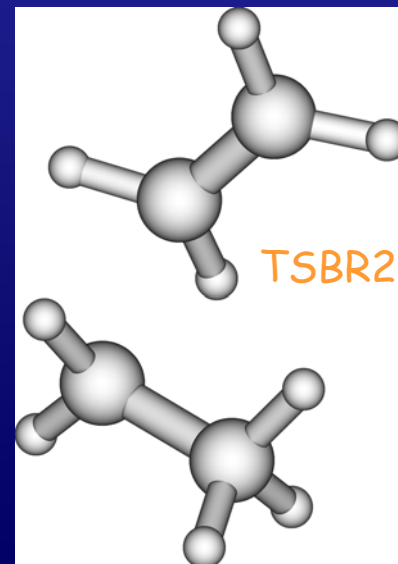
HO



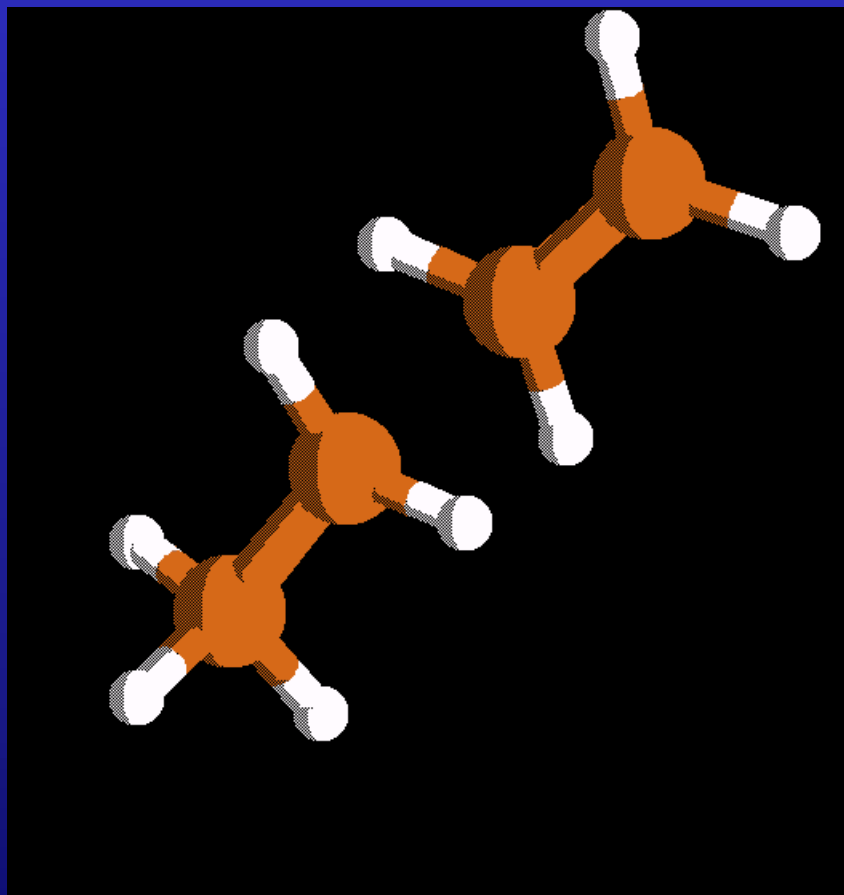
IR



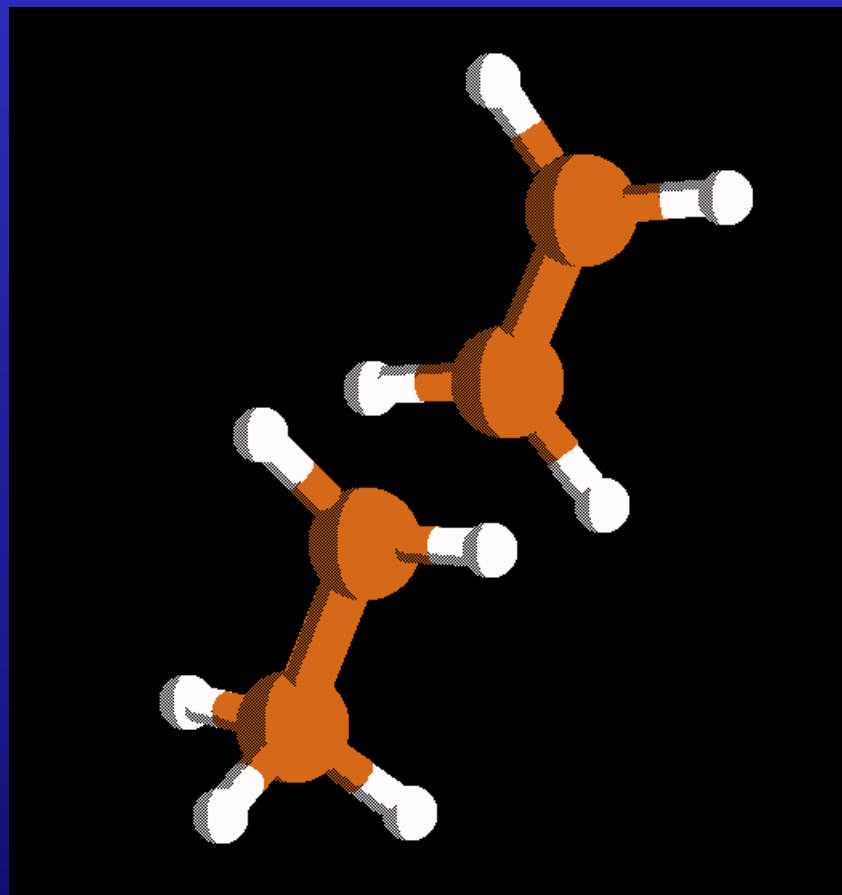
TSBR2

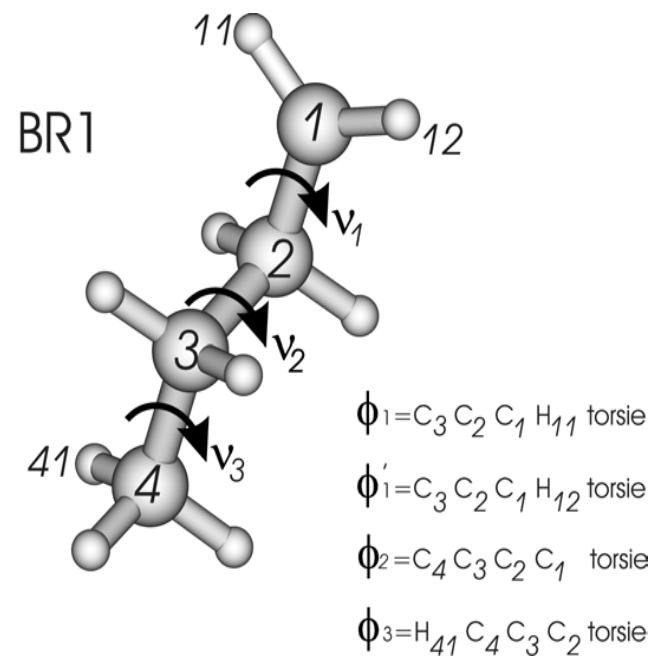
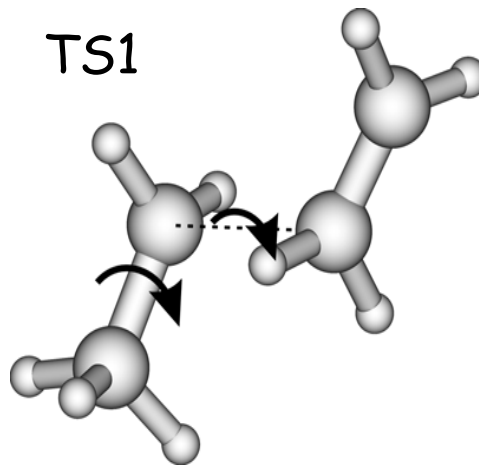
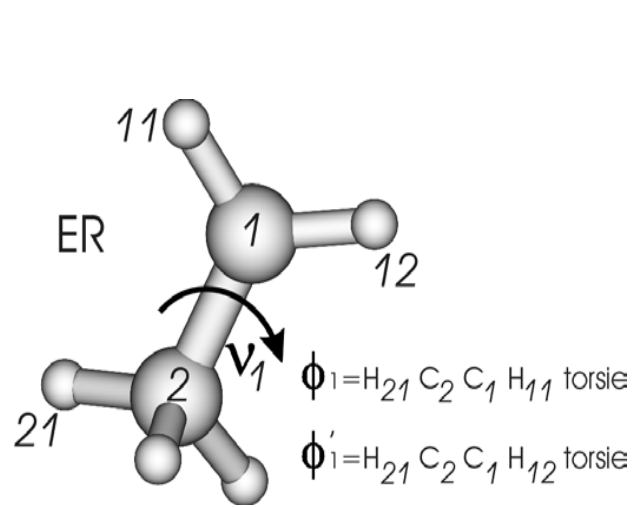


HO



IR





## Partition function $q$ :

$$q = q_{\text{trans}} q_{\text{vib}} q_{\text{rot}} q_{\text{electronic}}$$

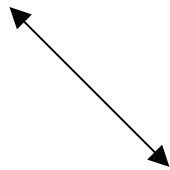


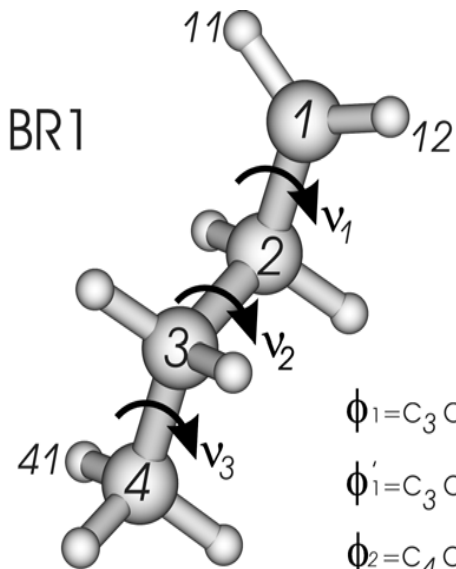
$$q_{\text{vib},i} = \frac{e^{-\frac{h\nu_i}{2k_B T}}}{1 - e^{-\frac{h\nu_i}{k_B T}}}$$

Some low vibrational modes correspond with internal rotations



$$q_{\text{rot,int},m} = \frac{1}{\sigma_{\text{int}}} \sum_k g_k(m) \exp\left(-\frac{\epsilon_k(m)}{k_B T}\right)$$





- $\phi_1 = C_3 C_2 C_1 H_{11}$  torsie
- $\phi'_1 = C_3 C_2 C_1 H_{12}$  torsie
- $\phi_2 = C_4 C_3 C_2 C_1$  torsie
- $\phi_3 = H_{41} C_4 C_3 C_2$  torsie

*In principle all internal rotations are coupled*

*Approximation : All internal rotations are decoupled with a rotational multidimensional rotational potential*

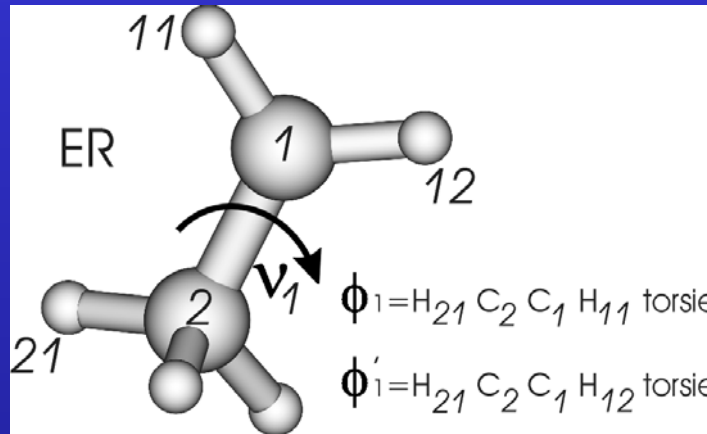
$$-\frac{\hbar^2}{2Im} \frac{\partial^2 \Psi_{km}(\phi_m)}{\partial \phi_m^2} + V(\phi_m) \Psi_{km}(\phi_m) = \epsilon_k^{(m)} \Psi_{km}(\phi_m)$$

*Reduced moment of inertia*

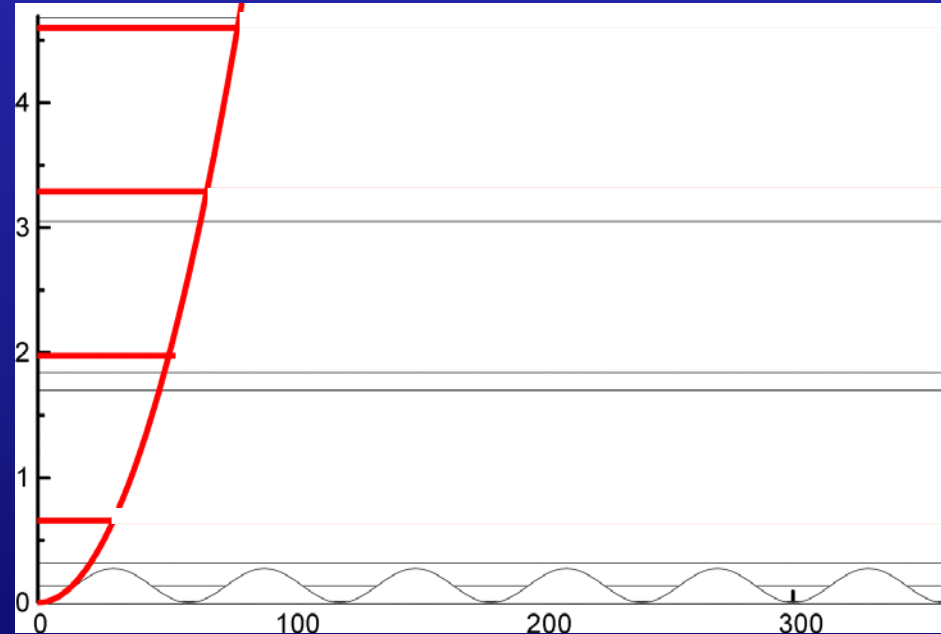
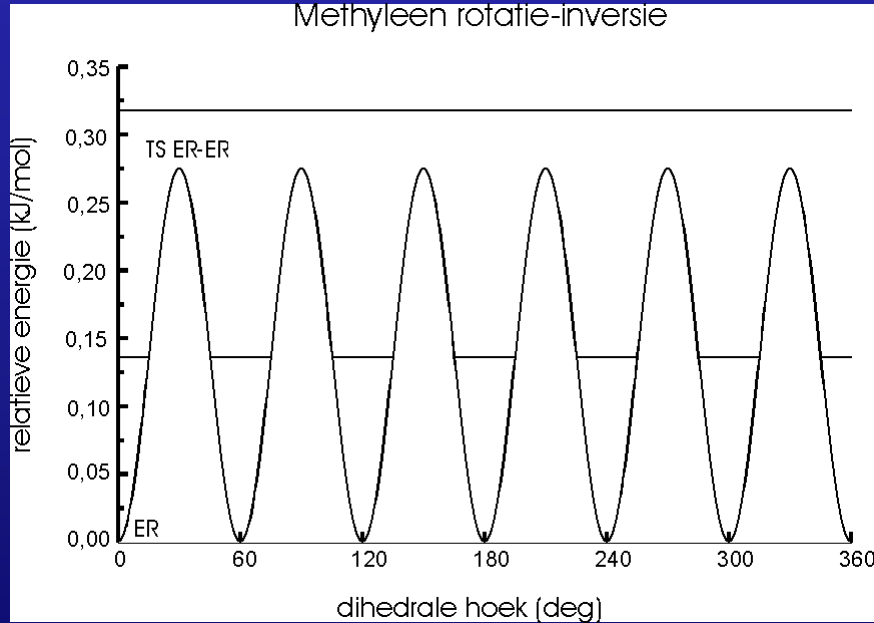
$$Im = Am \left( 1 - \sum_{I=X,Y,Z} \frac{Am \lambda_{mi}^2}{Ii} \right)$$

*Energy Eigenvalues*

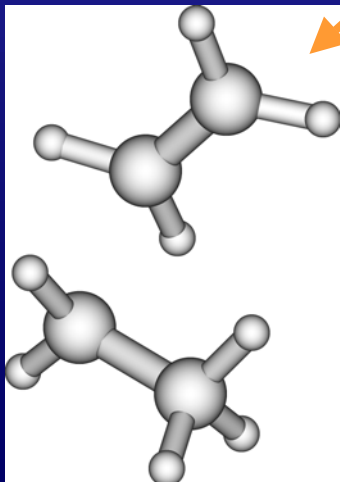
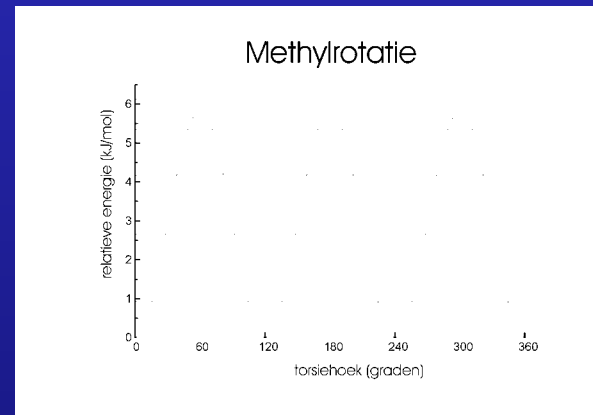
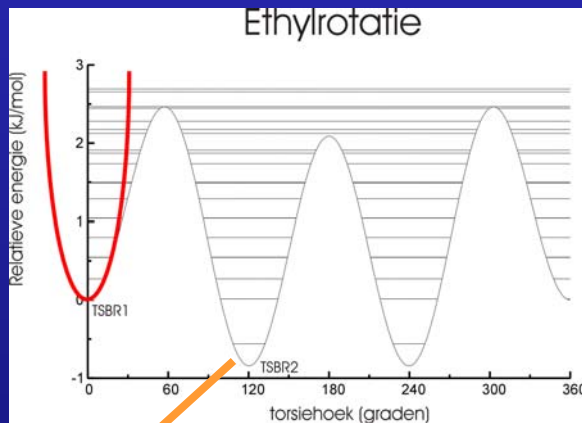
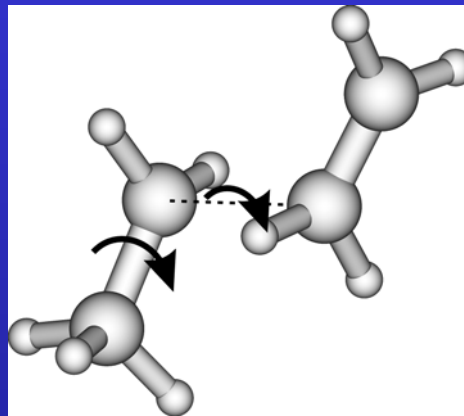
*Rotational wave function*



Methyleen rotatie-inversie

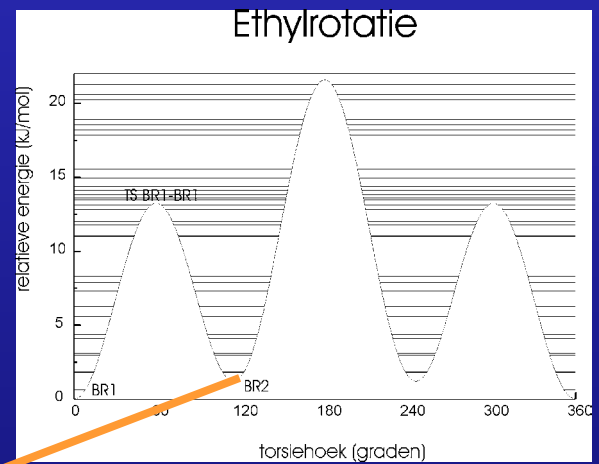
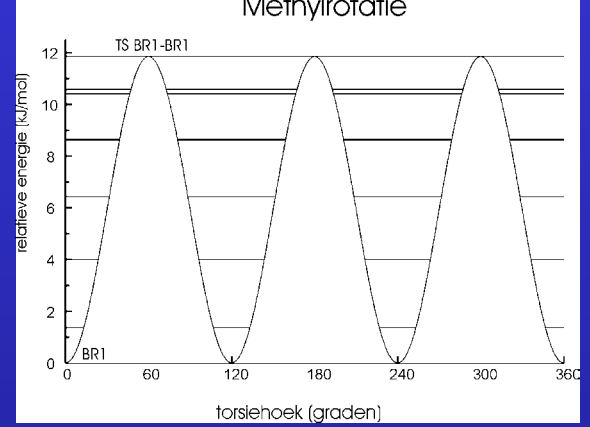
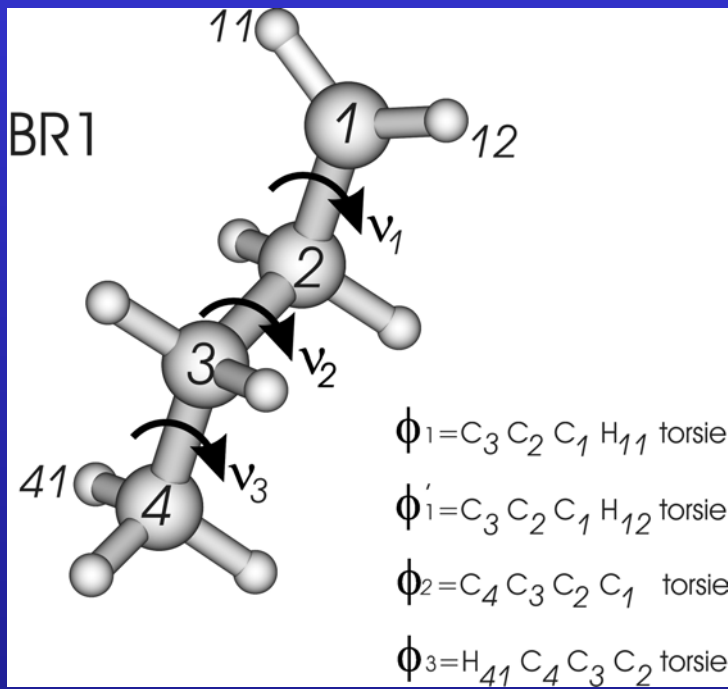


No extra conformers come into play,  
but the density of states increases,  
partition function increases

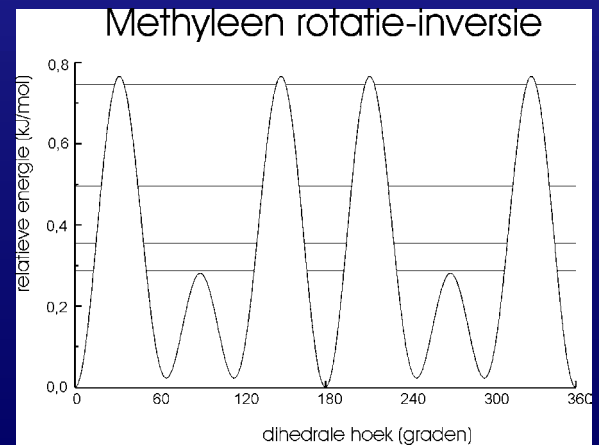
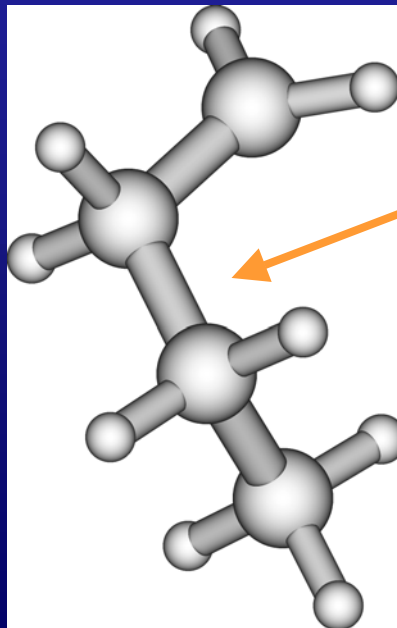


More conformers come into play

Addition reaction :  
gauche attack is energetically preferred



BR2





## Kinetic parameters with internal rotations for forward addition reaction

	$E_a$
HO	36.81
IR	37.06

Slight influence on activation  
energy

$A$
$9.8 \text{ E}7 \text{ dm}^3/\text{mol}\cdot\text{s}$
$9.8 \text{ E}8 \text{ dm}^3/\text{mol}\cdot\text{s}$

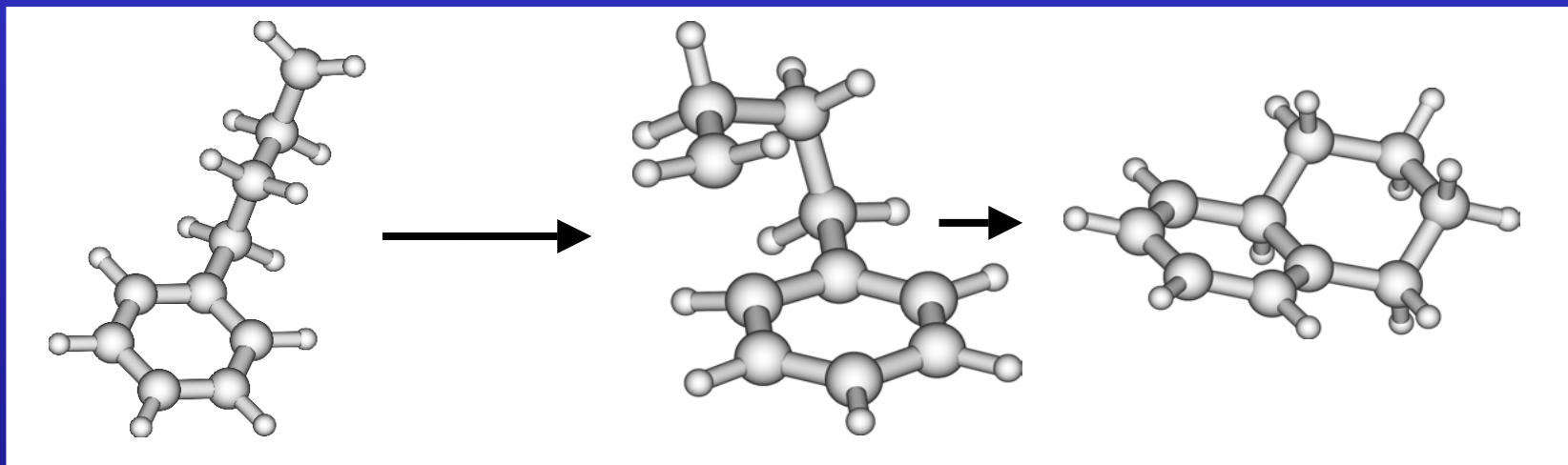
$$\frac{A_{\text{IR}}}{A_{\text{HO}}} = 10$$

- More conformers
- Density of states increases

## Outline

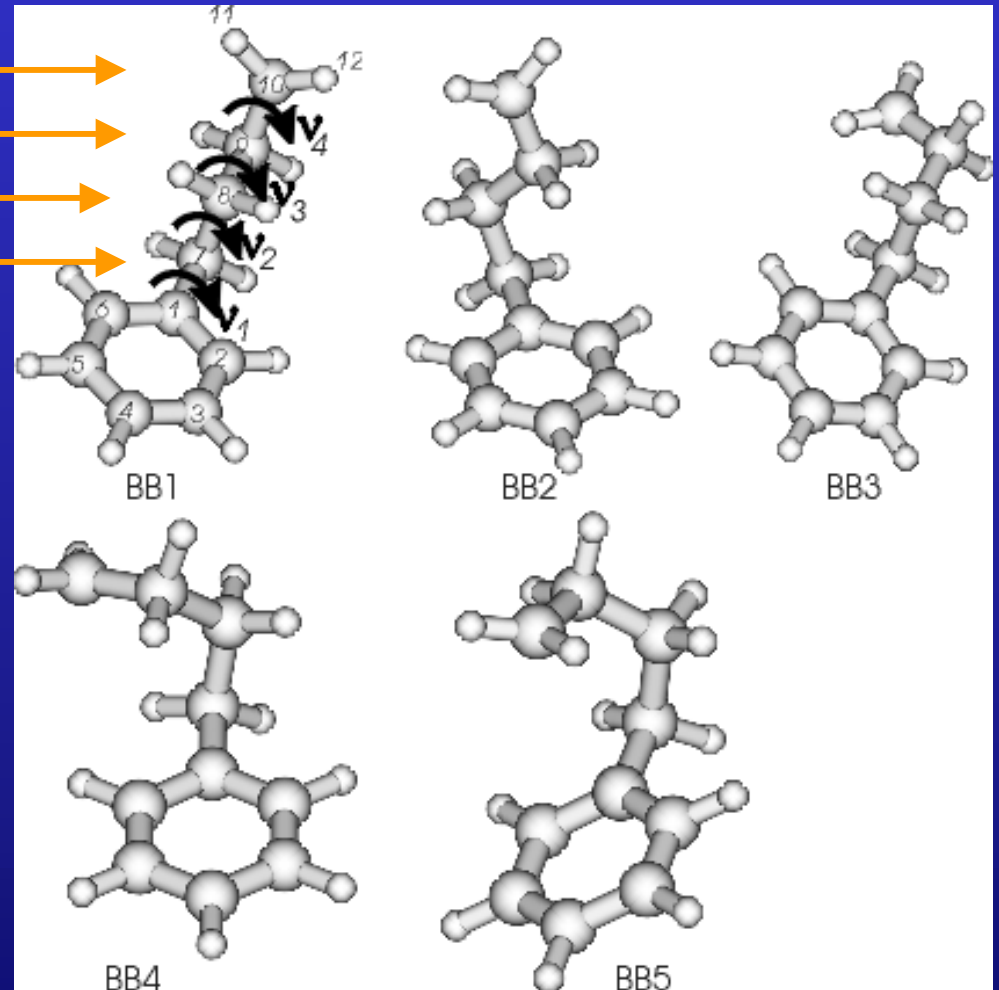
- Theoretical procedures on addition of ethylradical to ethene
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- Hydrogen abstraction reactions : Free rotor approach
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## Cyclization of the butylbenzene radical



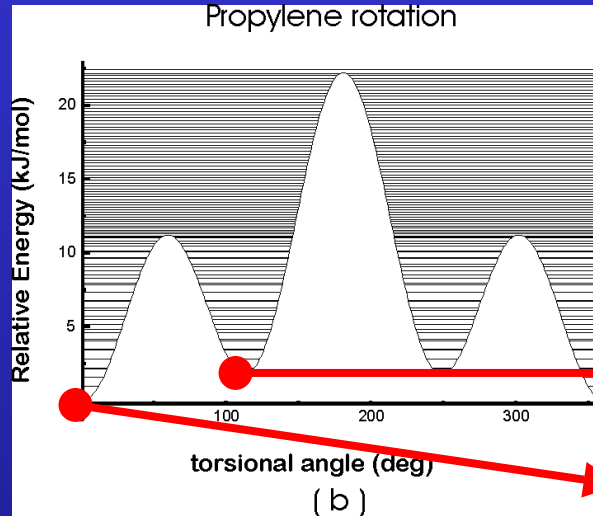
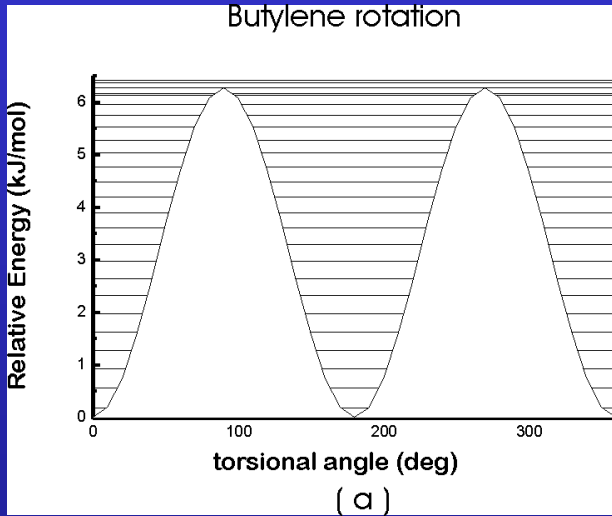
More internal rotations must  
be activated to reach the transition state

methylene rotation-inversion  
ethylene rotation  
propylene rotation  
butylene rotation



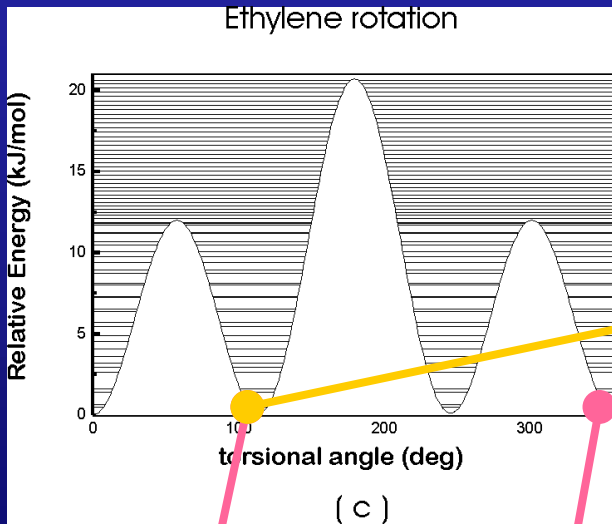
### *Conformational Analysis :*

- *orthogonal conformation of butylchain*
- *combinations of gauche and anti orientations of CCCC torsional angles*



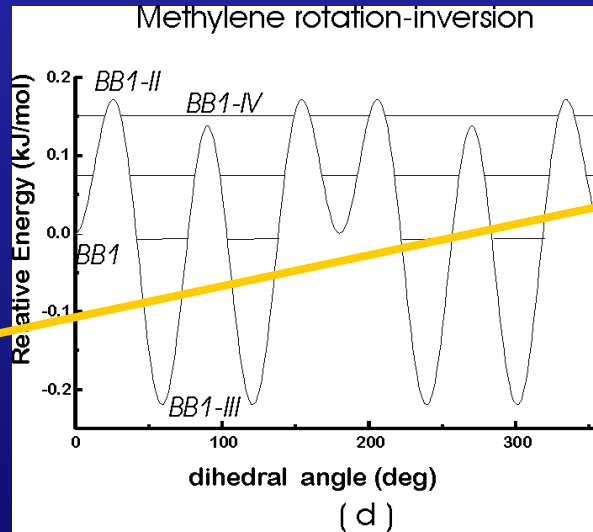
BB2

BB1



gauche

trans



BB3

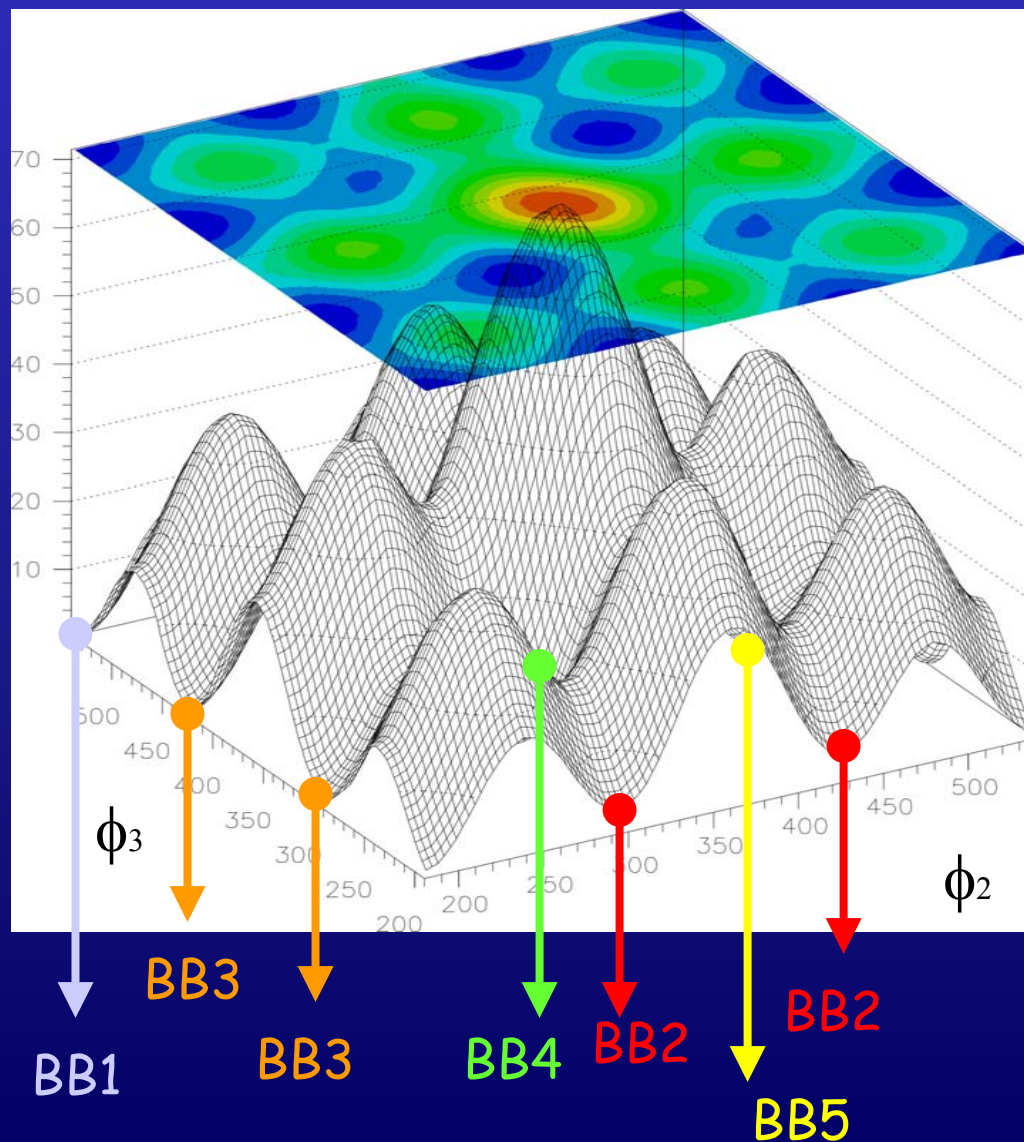
All conformers that can be reached by one internal rotation are taken into account correctly

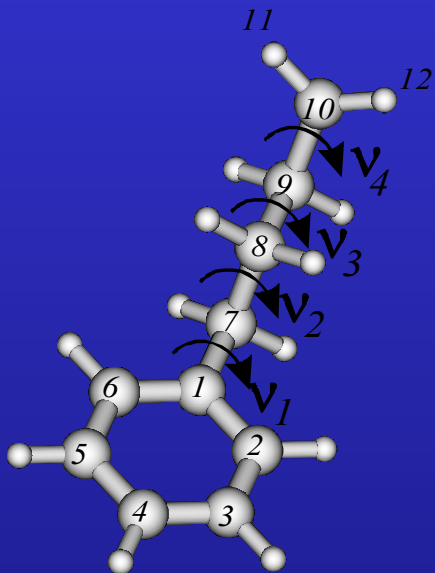
How are other conformers like BB4, BB5 taken into account ?

Some conformers are only accessible by a path on the 2-dimensional PES



Correctly described by coupled internal rotations





*The global and all internal rotations are coupled*

Rotational Hamiltonian :  $H = H_R + H_T + H_{RT}$

Global rotation

Internal rotation

Coupling between total and internal rotation

Coupled internal rotations

$$H_T = \frac{1}{2} A J_2^2 + \frac{1}{2} B J_3^2 + C J_2 J_3 + V(\phi_2, \phi_3)$$

$$\downarrow \qquad \qquad \downarrow$$

$$-\hbar^2 \frac{\partial^2}{\partial \phi_2^2} \qquad -\hbar^2 \frac{\partial^2}{\partial \phi_3^2}$$

$$\bullet \longrightarrow \Psi_k(\phi_2, \phi_3)$$

$$\varepsilon_k$$

Uncoupled internal rotation

$$V(\phi_2, \phi_3) \bullet \longrightarrow V(\phi_2) + V(\phi_3)$$

$$-\frac{\hbar^2}{2I_2} \frac{\partial^2 \Psi_k(\phi_2)}{\partial \phi_2^2} + V(\phi_2) \Psi_k(\phi_2) = \varepsilon_k \Psi_k(\phi_2)$$

# Influence of coupled internal rotations on partition functions

Ethylene and propylene internal rotation in butylbenzene radical

$q_{HO}(300K) =$

$q_{IR,uncoupled}(300K) =$

$q_{IR,coupled}(300K) =$

ethylene    propylene

2.38

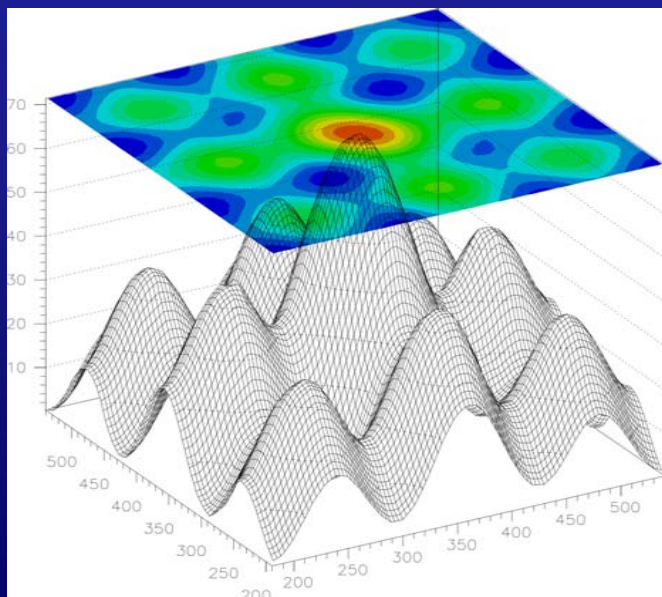
3.00

7.71

\* 7.83 = 60.36

23.93

Density of states is larger in IR rotor case versus HO case  
More conformers can be accessed



Partition function decreases, due to large bump in potential energy surface, energy levels are shifted up



## Influence of coupled internal rotations on the kinetic parameters

### Cyclization Reaction :

$$E_a(\text{HO}) = 49.53$$

$$E_a(\text{IR,uncoupled}) = 49.25$$

$$E_a(\text{IR,coupled}) = 49.21$$

$$A(\text{HO}) = 1.5\text{E}10$$

$$A(\text{IR,uncoupled}) = 1.6\text{E}9$$

$$A(\text{IR,coupled}) = 3.0\text{E}9$$



Activation energy is primarily determined by difference in groundstate energies between transition state and reactant

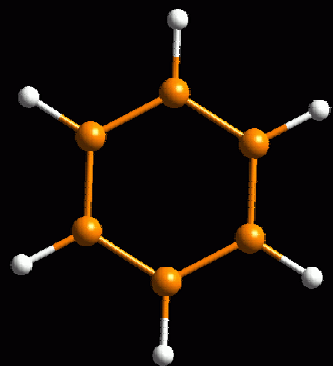
Influence of coupled internal rotations is rather small on  $A$  for this particular reaction

Work is in progress to test the approach on other reaction and on other thermodynamic quantities

## Outline

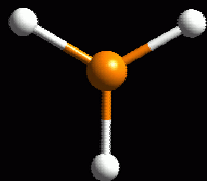
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# Hydrogen abstraction reactions



Benzene

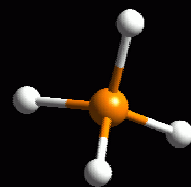
+



methylradical

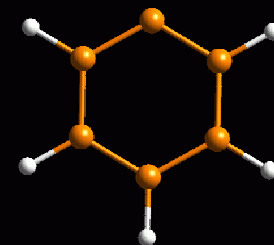


ts



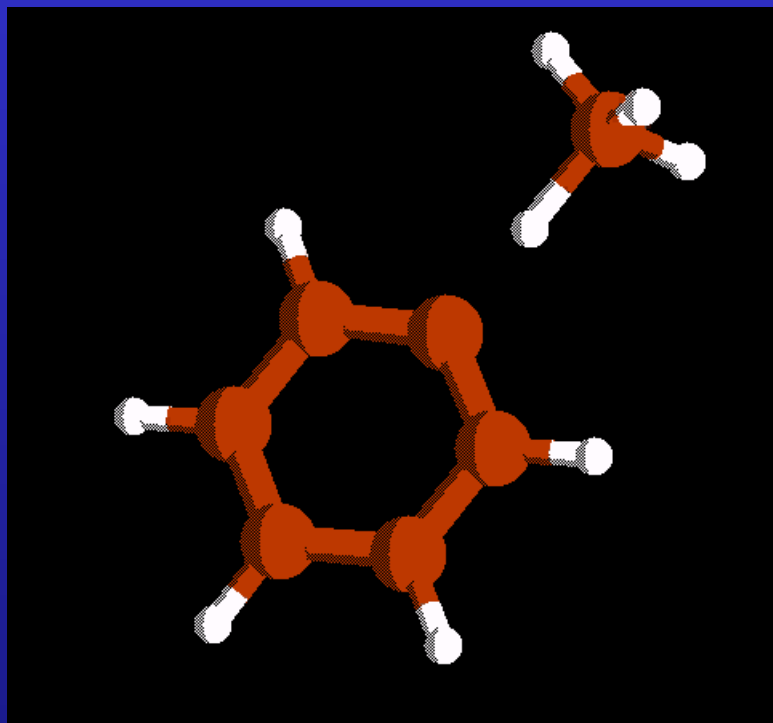
Methane

+

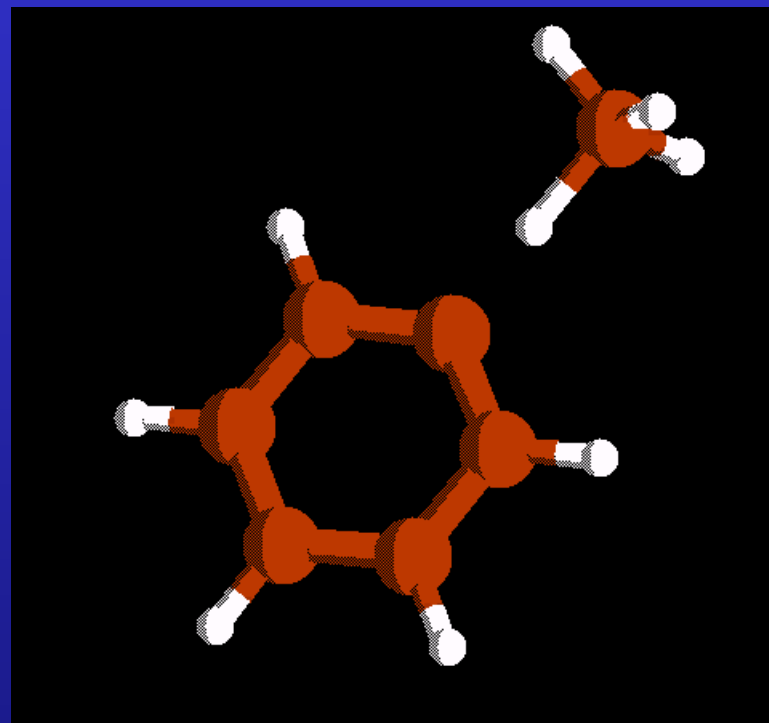


Phenylradical

One imaginary frequency corresponding with reaction coordinate



Internal rotation with very loose potential  
Frequency 8.39 1/cm



Standard from ab initio packages : All vibrations in HO approximation

$$\begin{array}{l} q_{\text{vib1}}(300\text{K})_{\text{HO}} = 25.8 \\ q_{\text{vib1}}(300\text{K})_{\text{Free Rotor}} = 11.1 \end{array}$$

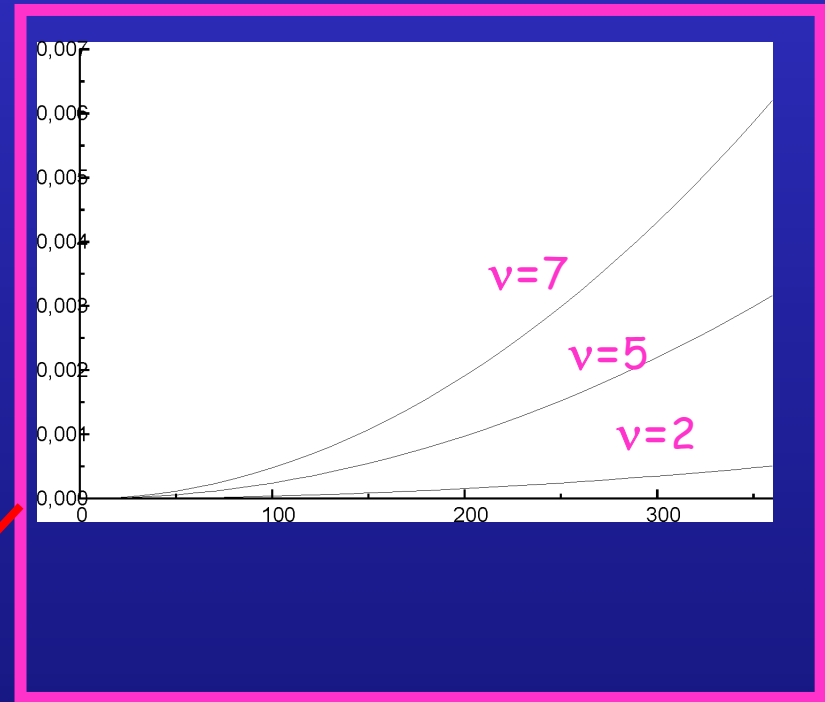
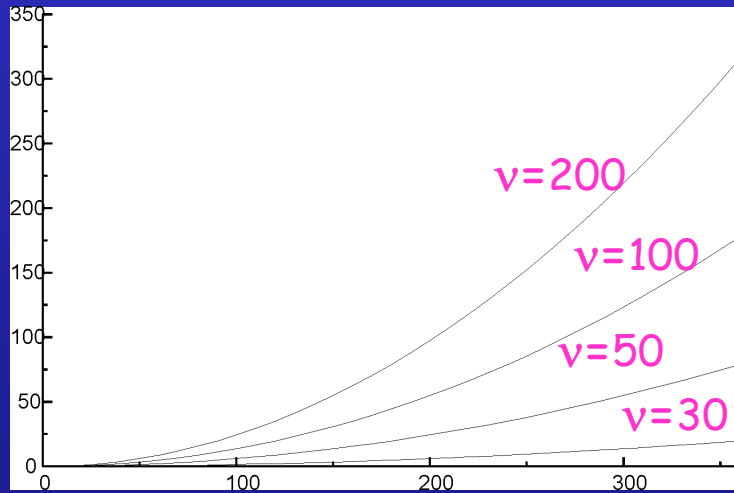
HO description is not physical in this case

# HO description :

$v$

force constant :  $k = I m \omega^2$

HO potential :  $V = 0.5 * k * \theta^2$



Describe as free rotor

Not a realistic representation as HO.  
*HO description is only valid  
if the potential causes serious hindering*

Partition function of transition state :

$$q(\text{HO}, 700\text{K}) = 60.26$$

$$q(\text{Free rotor}, 700\text{ K}) = 17.04$$

Kinetic parameters :

$$E_a(\text{HO}) = 79.68 \text{ kJ/mol}$$

$$E_a(\text{free rotor}) = 76.05 \text{ kJ/mol}$$

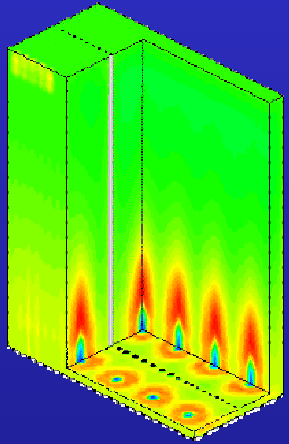
$$A(\text{HO}) = 3.36 \text{ E}9 \text{ m}^3/\text{mol s}$$

$$A(\text{free rotor}) = 5.14 \text{ E}8 \text{ m}^3/\text{mol s}$$

For hydrogen abstractions, serious overestimation of partition functions of transition state in HO approximation

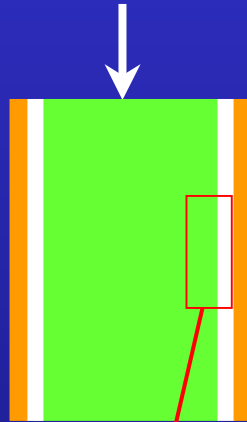
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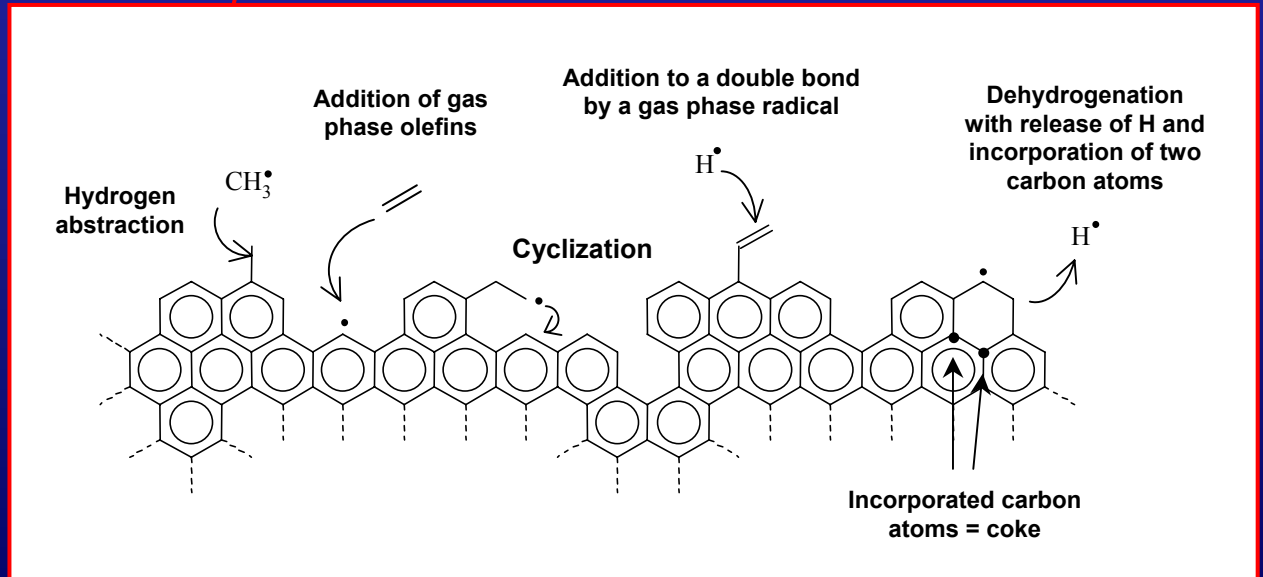
3-dimensional view of a thermal cracking furnace

Process gas :  
Hydrocarbon mixture



Tube skin in Cr/Ni/Fe alloy

Coke layer





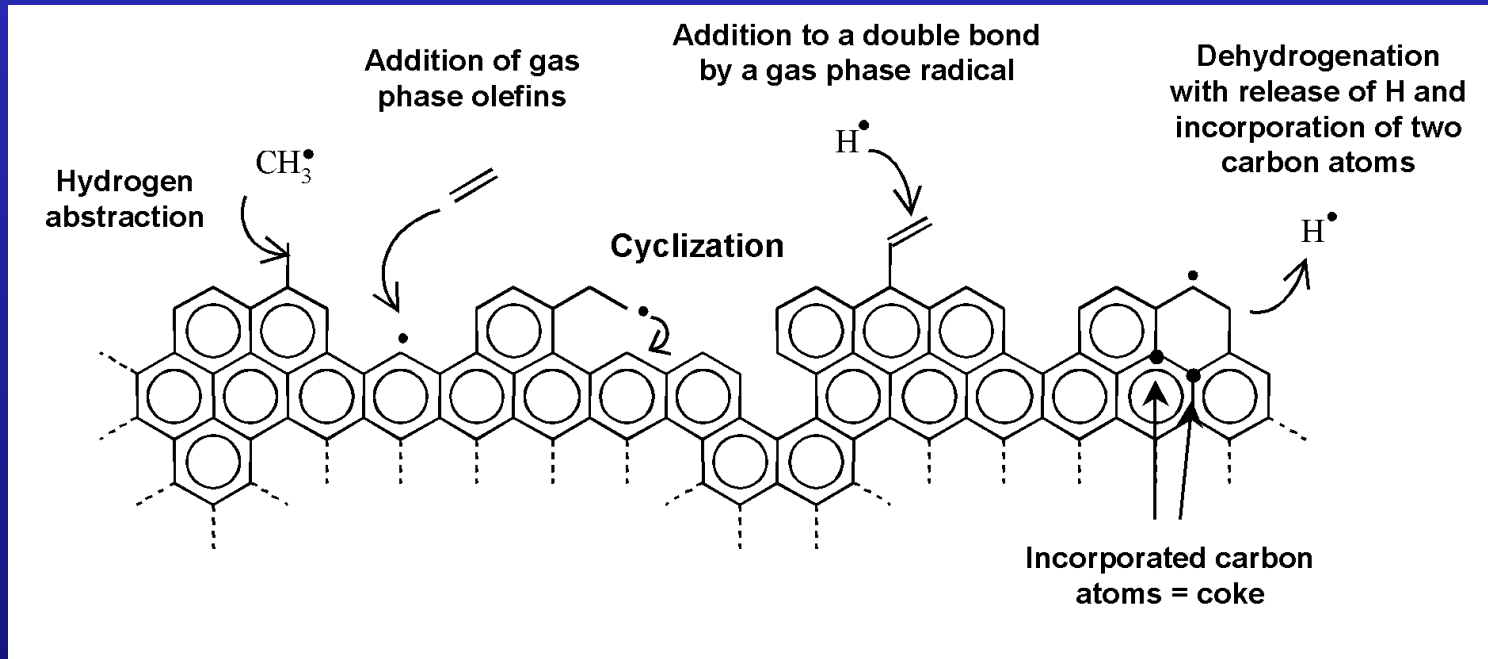
Thermal cracking network with more than 1000 reactions

?

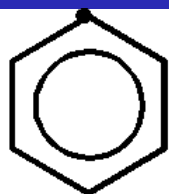
Information on kinetic and mechanistic aspects  
of elementary reactions of the network

*Ab initio* calculations on elementary reactions of the network

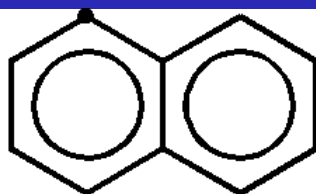
# Elementary Reactions



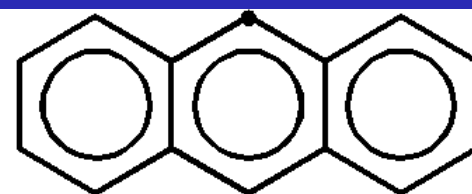
Influence of the local coke matrix on the kinetic parameters.



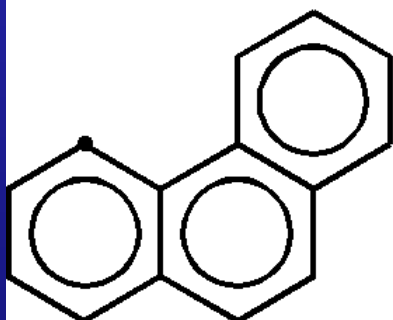
BENZENE



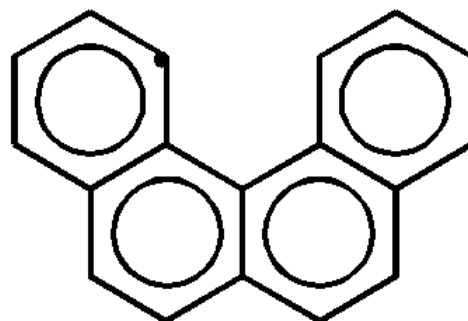
NAPHTALENE



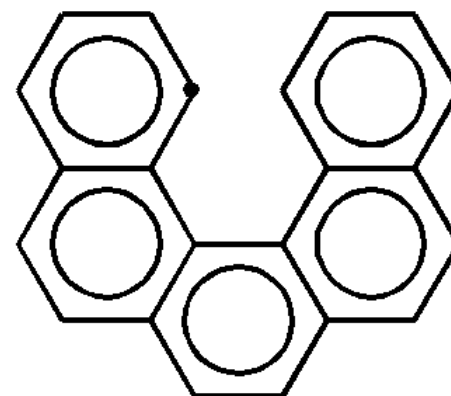
ANTHRACENE



PHENANTRENE

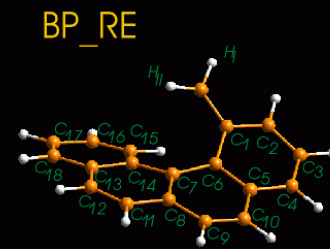
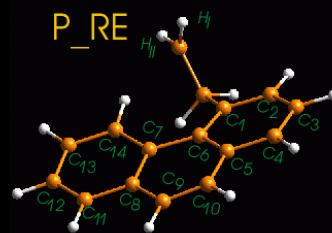
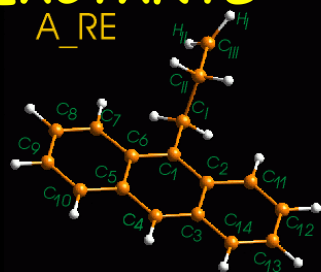
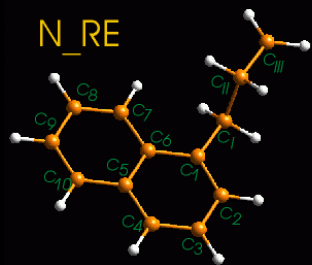
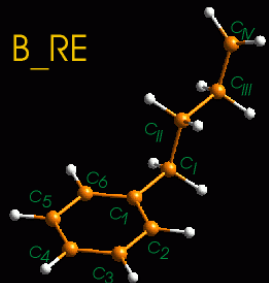


BENZOPHENANTRENE

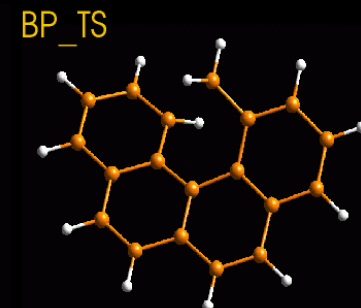
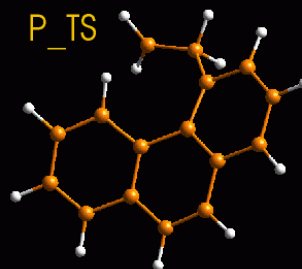
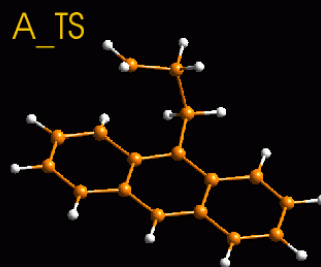
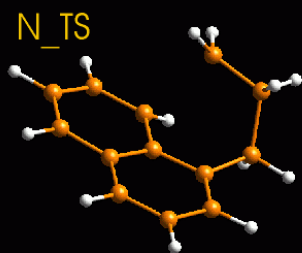
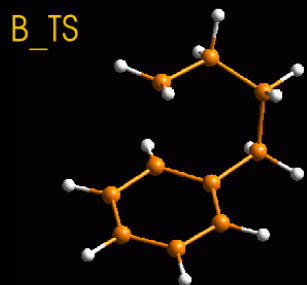


DIBENZO(C,G)PHENANTRENE

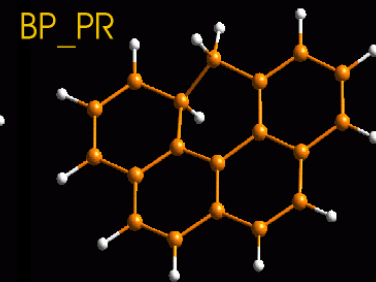
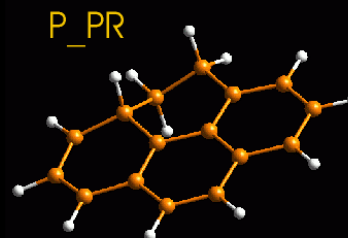
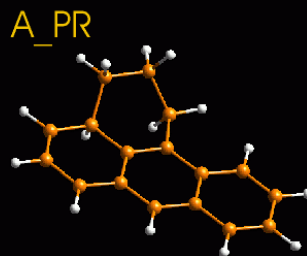
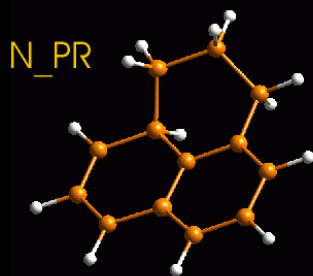
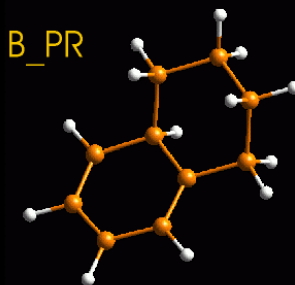
## REACTANTS

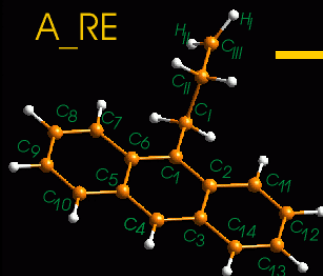
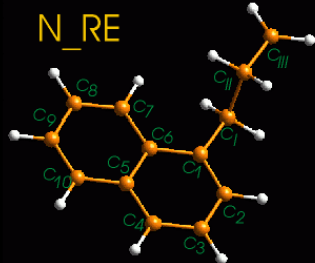
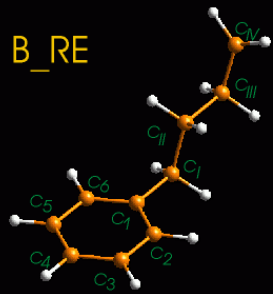


## TRANSITION STATES

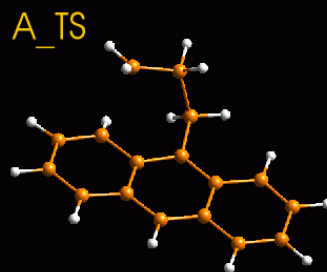
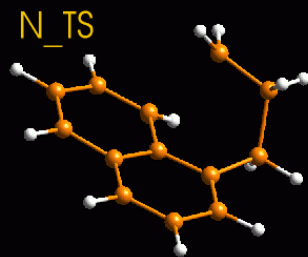
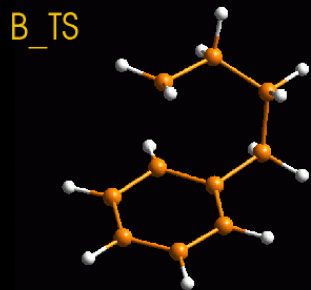


## PRODUCTS

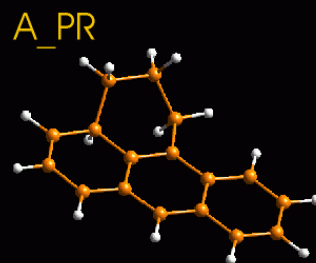
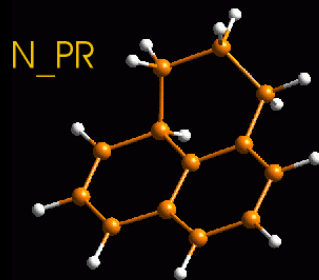
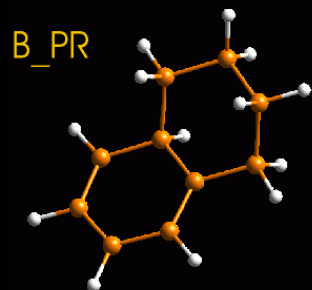




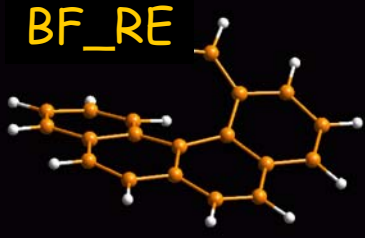
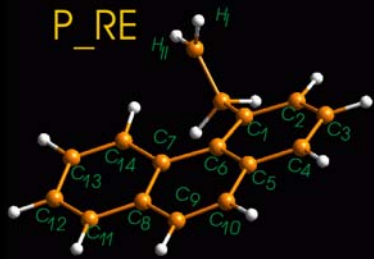
Aromaticity is preserved in reactant



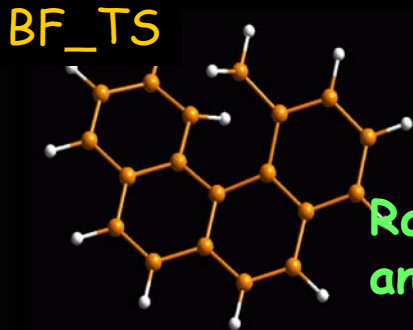
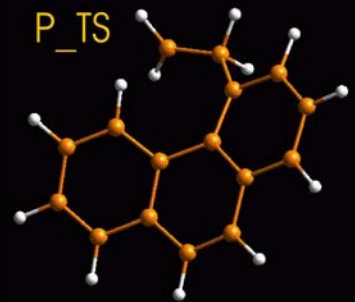
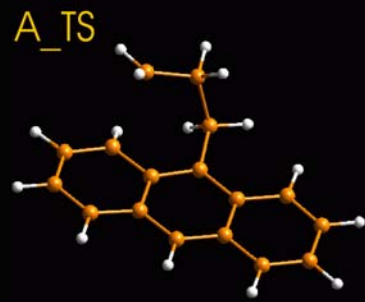
Transition state is stabilized by resonance stabilization



	$E_a$	$A$
B	49.53	1.5 E10
N	47.79	1.3 E11
A	40.38	2.4 E11

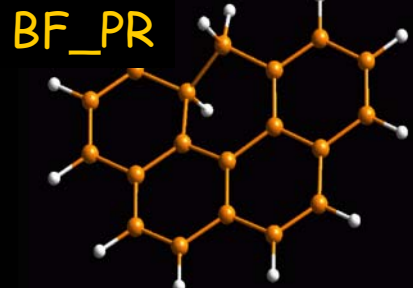
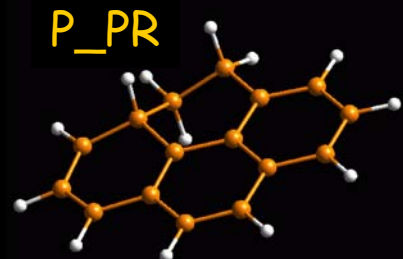
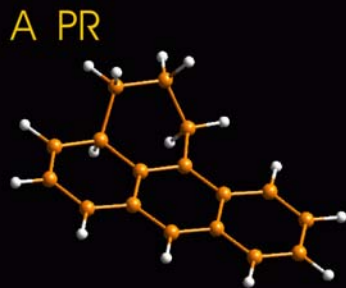


Aromaticity is partly broken in reactant



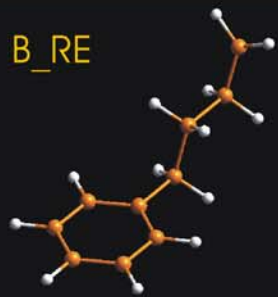
Substrate is more reactive  
"Folded cluster"

Radical is of the benzylic type and less reactive

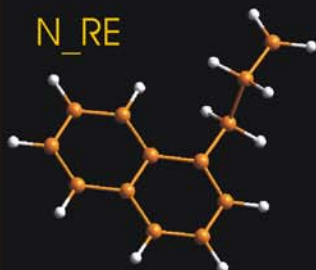


	Ea	A
A	40.38	2.4E11
P	32.64	3.8E11
BP	50.35	1.9E12

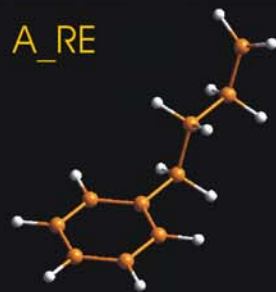
## Preexponential factor



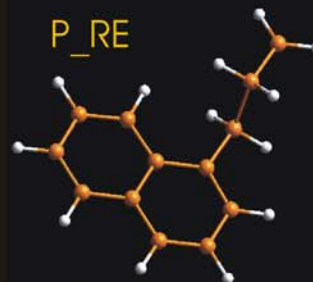
1.5 E10



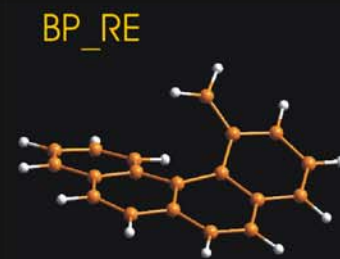
1.3E11



2.4E11



3.8E11



1.9E12

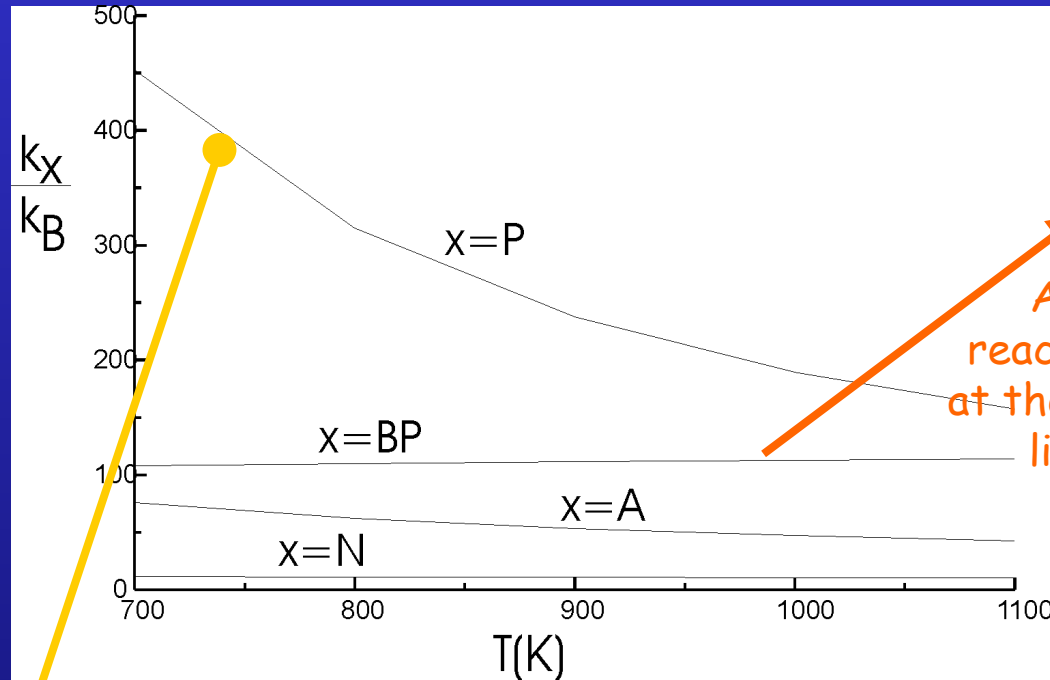


The alkylchain is longer, the conformational flexibility increases

The probability that a the reactant molecule resides in a conformation suitable for cyclization decreases

Not a pure effect of the structure of the coke matrix

## Influence of the local structure of the coke matrix on the reaction kinetics



At the end of the reactor also cyclization at the benzophenanthrene-like site becomes important

← Temperatures in a typical cracking unit 700K-1100K →

Ring closure at "folded" clusters is much faster than at cluster in which aromaticity is preserved in reactant.

Coke formation is autocatalytic : Larger macroradicals, react faster than the smaller ones they originate from



## Conclusions

- Rate constants can be calculated from first principles
- Microscopic insight into the factors that govern reaction barrier and preexponential factor
- Influence of correct theoretical description of internal rotations is large

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